

Consumer Privacy and Targeted Pricing with Stochastic Valuations

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Abstract

I study the market for consumer information. Firms cannot commit to privacy, so buyers anticipate disclosure of purchase history for targeted pricing. In the literature, strategic rejections of informative offers make purchase data worthless, which is paradoxical given the large investments in data analytics. I show that buyers' uncertainty regarding future preferences allows full separation of types, leading to valuable purchase data. Buyers of any given type are assumed to share similar expectations about the evolution of future valuations. Preference uncertainty generates consumer-to-consumer externalities, which is why strategic consumers are sometimes better off when they remain unaware of targeted pricing. When preferences are transitory, firms have an incentive to raise consumer awareness about targeted pricing.

Keywords: Consumer Privacy, Purchase History, Strategic Rejections, Strategic Purchases, Targeted Pricing

1. Introduction

Innovations in technology enable businesses and information intermediaries to collect extensive records of consumer behavior. “Data brokers” collect, and merge, raw and fragmented data – from online and offline sources – and develop products used in a variety of business applications including marketing, predictive analytics, and fraud detection.¹ By inferring preferences from past behavior, firms can offer product recommendations and optimize bidding algorithms for position or placement auctions for targeted advertising.

Enterprises in the emerging “personal data vault” industry (*e.g.*, Personal.com and Mydex.org) provide cloud-based services such as secure access to subscribers' data, automatic form filling, and identity verification. Subscribers also choose when private information, such as *intended* purchases, may be disclosed to third parties. Though beneficial, these innovations also raise concerns of consumer privacy and business practices related to the collection and use of consumer data.²

In this paper we study the use of purchase history for *targeted pricing*. Though the analysis is relevant for targeted advertising, we do not cover it explicitly. It is well known that targeted pricing is generally *not* profitable when buyers are strategic and have preferences that are constant

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¹The New York Times (2014) reports some health insurers are using information from data brokers on income, education, race, online shopping habits *etc.*, to refine risk-forecasting models.

²The U.S. Federal Trade Commission recently released a report, FTC (2014), on the data-broker industry, which includes recommendations to improve transparency and awareness.

or positively correlated across time. The information rent necessary to induce separation between types overwhelms any gain from dynamic pricing.³ Thus, whenever possible, firms are better off committing to full privacy, as shown by Calzolari and Pavan (2006). When firms *cannot* commit to nondisclosure, then – despite reduced profit – one would still expect consumer purchase histories to be valuable. However, as the literature shows, purchase histories have no value with rational consumers. Acquisti and Varian (2005) find this paradoxical, especially given the large investments businesses make in collecting and analyzing consumer data.

We present endogenous conditions under which purchase history is valuable with rational buyers. We do not rely upon prior assumptions made in the literature such as; consumer naïvety, complementarity in products, direct externalities between upstream and downstream firms, quasi-*convex* utility functions or differences in discount factors. Instead, we make a simple and purely informational assumption; buyers are *uncertain* about their future demand. It is easy to think of examples where outside options evolve stochastically.⁴

Our results suggest that consumer data can be more valuable when consumers are *less certain* about future preferences. In another surprising result, consumers are sometimes better off when they remain oblivious of targeted pricing, a result we attribute to externalities strategic buyers impose on each other and inefficient signaling. These results do not arise when preferences are deterministic. Furthermore, unlike the literature, *all buyers* have strategic significance.

The metaphor used in the analysis is one of buyers making sequential purchase decisions from two firms. Buyers have unit demand in each period and their valuation for any firm’s product can be either high or low. Buyer valuations evolve as a two-state Markov process. This has two important implications: (i) buyers do not know their future valuations in advance and (ii) all buyers – of a given type – have similar expectations about the evolution of future valuations. These assumptions lead to results that differentiate the current work from the literature.

Targeted pricing is implemented in two stages. In the first stage firms engage in price experimentation, which may reveal buyer preferences. Dynamic pricing follows later, based on buyers’ prior actions, and the expected evolution of preferences. Depending on the second firm’s predisposition this takes the form of either *targeted discounts* or *targeted markups*. Dynamic pricing is always profitable with unaware consumers. However, when rational consumers anticipate disclosure, their inevitable strategic response alters the profitability and welfare consequences of targeted pricing.

Uncertainty in future preferences leads to bounds on first-period prices that buyers cannot accept or reject with certainty. These thresholds make full separation feasible. Targeted pricing becomes unprofitable when at least one type of buyer has *persistent* preferences or, more precisely, when the sum of the two persistence probabilities in the Markov process exceeds one (*Condition A*, in the paper). In this case, buyers’ incentives to conceal, or *reveal*, private information lead to lower prices, leaving firms worse off. Lacking the ability to commit to nondisclosure, firms have a strong incentive to keep buyers unaware of targeted pricing. Unlike the literature, consumer data are valuable for a large set of parameter values.

Things are strikingly different when at least one type of buyer has *transitory* preferences or, specifically, when the sum of persistence probabilities is less than one (*Condition B*). Now, incentives to conceal or reveal private information get reversed and, at times, buyers are even willing to pay more than their reservation values. This allows firms to raise prices. Under these conditions, firms are better off with targeted pricing, and have an incentive to *raise buyer awareness* about the same. Again, consumer data are valuable for a large set of parameter values.

³In this paper, the terms behavior-based pricing, dynamic pricing, and targeted pricing are all interchangeable.

⁴During periods of recessions or monetary tightening, credit-constrained individuals are likely (but not guaranteed) to remain so in the next period (positive serial correlation in outside options). However, others not previously constrained may become so with high probability (negative correlation). The opposite may happen during expansions.

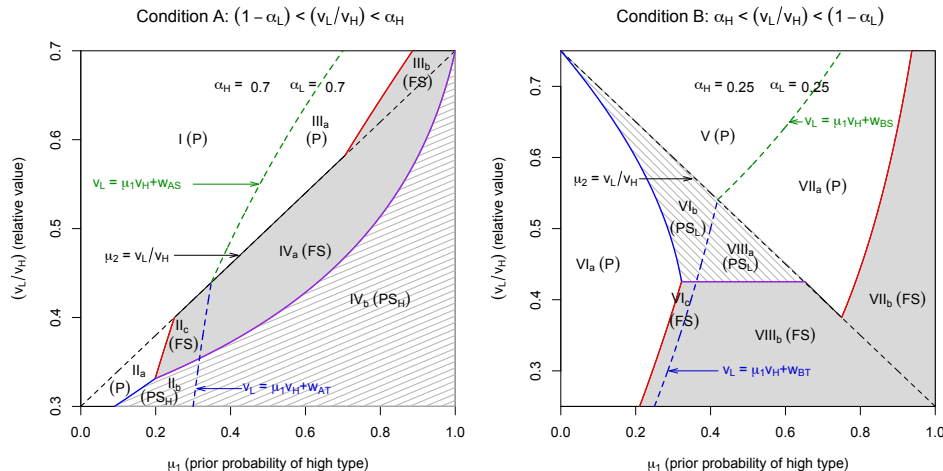


Figure 1: Regions with valuable purchase data (solid grey)

Figure 1 provides an early look at equilibrium outcomes in the first stage. A fuller description of these illustrations appears later. In these illustrations, persistence probabilities are chosen exogenously.⁵ The left panel corresponds to a situation with positive serial correlation in buyer valuations, and the right panel provides an example with negative serial correlation in buyer preferences. Condition A (left panel) or B (right panel), whichever is relevant, leads to *endogenous* bounds on the ratio of buyer valuations, plotted on the y -axis. The initial fraction of high-value buyers is on the x -axis.

Solid grey regions represent fully separating outcomes (FS), which are associated with valuable consumer data. Regions with shading lines represent partially separating equilibria (PS_H or PS_L), where some buyers randomize their purchase decision. Partial separation supports targeted pricing, but purchase histories are not valuable as they do not increase downstream profit. Regions without any shading represent pooling (P) in the first period. We use these regions to point out similarities and draw contrasts with the literature.

2. Related Literature

The literature on privacy and dynamic pricing is vast. We compare our results with a few papers relevant to our main interest, which is to explain – with minimal assumptions – how consumer data can be valuable with rational buyers.

Taylor (2004), and Acquisti and Varian (2005) show that purchase history is valuable when buyers do not anticipate dynamic pricing (‘naïve’ or uninformed buyers), but not when they are informed. Our assumption of stochastic preferences leads to valuable consumer data even with informed consumers. Like Taylor, we allow buyer valuations to invert between periods. However, unlike Taylor, we assume that buyers do not know their future valuations in advance.

In Taylor (2004), only those buyers who *know* they have the high valuation in both periods have strategic significance. The implication is that Taylor’s *outcomes* are similar to the setting with constant types, as in Acquisti and Varian (2005). With rational buyers, only three outcomes can arise in equilibrium, *none* of which support valuable purchase history. These equilibrium outcomes are a subset of our own, and correspond to regions I , III_a , and IV_b in Figure 1. Equilibria in region

⁵In the analysis, persistence probabilities of the two buyer types (denoted by α_H and α_L) are allowed to be distinct and completely arbitrary, *i.e.*, any value between 0 and 1.³

I involve pooling at the low price in both periods. Region III_a also involves pooling in the first period, but the second firm offers just the high price. Region IV_a involves both firms offering the high price, with only partial separation of high-value buyers. Without fully separating equilibria, consumer data have no value. Our results for Condition B cannot arise in these environments as they do not consider negative correlation.

Loginova and Taylor (2008) relax Taylor’s (2004) assumption that buyers know their valuations in advance. Even with this relaxation, they find that fully separating equilibria are unlikely. The main difference in the informational assumptions is that, in the current paper, all buyers – of a given type – have similar expectations about the future evolution of preferences. In Loginova and Taylor (2008), even if a buyer’s initial valuation is revealed, there is a lot of heterogeneity in expectations about future valuations, which makes purchase data less valuable.⁶

Using a mechanism design approach, Calzolari and Pavan (2006) show that information disclosure *can be* optimal in three cases: (i) buyer preferences exhibit perfect negative serial correlation, (ii) upstream and downstream products are complements (or substitutes), or (iii) the upstream firm has a direct interest in the downstream level of trade (direct externalities). It is intuitive that disclosed information is valuable when the goods are complements or with direct externalities; we do not consider either. But the first case is relevant. Information disclosure is optimal in Calzolari and Pavan (2006) when valuations are perfectly negatively correlated. However, this does not imply that the disclosed information has value. Rather, the incentive to disclose information is through a rent-shifting effect, and firms receiving information do not pay anything for it. The corresponding outcome in our case appears in regions VI_b and $VIII_a$ in Figure 1.⁷

The right panel of Figure 1 presents a scenario in which the persistence probabilities for both types of buyers is 0.25 (negative serial correlation). As these probabilities drop to zero, we approach the *perfect* negative correlation case of Calzolari and Pavan. When this happens, the fully separating, solid grey regions (VI_c , VII_b , and $VIII_b$) collapse, and the result coincides with Calzolari and Pavan. Staying with negatively correlated values, Calzolari and Pavan find that disclosure is not optimal when the second firm has beliefs *favorable* to the buyer, situations where the downstream firm has a predisposition to offer the low price to all. However, as explained later, regions V , VII_a , and VII_b suggest otherwise. Even a small amount of uncertainty in valuations leads to valuable consumer data, and disclosure is always optimal with negatively correlated valuations.

There are other explanations for valuable consumer data. Acquisti and Varian (2005) show that purchase history can be valuable if buyer valuations exhibit *increasing* marginal utility. Then, in addition to prices, firms also condition the quality of service on purchase history. They also show that consumer data may be valuable when buyers do not have easy access to *anonymizing* technologies.⁸ Bikhchandani and McCardle (2012) show that when sellers are more patient than buyers, the optimal dynamic contract is not a repetition of the static contract suggested in Baron and Besanko (1984). Instead, purchase may result in higher prices. This is related to a result from Fudenberg and Tirole (1983), in a setting without commitment, where *buyer impatience* drives a wedge between fully and partially separating equilibria.

Chen and Zhang (2009), and Shin and Sudhir (2010) study targeted pricing in dynamic

⁶In Taylor (2004) and Loginova and Taylor (2008) valuations arise as repeated draws from a Bernoulli distribution with a privately known, and uniformly distributed, ‘demand’ parameter. Thus, there are infinite types of buyers. The (uniformly distributed) Bernoulli assumption implies a lot of heterogeneity, and also rules out negative correlation.

⁷Calzolari and Pavan (2006) allow optimal mechanisms to be stochastic. This is crucial in mapping our mixed-strategy results to probabilistic mechanisms. Randomized purchases, in this paper, correspond to lotteries for trade to occur in their setting.

⁸Readers interested in the impact of anonymizing techniques, *e.g.*, ‘private’ browsing sessions, or *marketing avoidance* are directed to Hann, Hui, Lee and Png (2008), and Conitzer, Taylor and Wagman (2012).

duopolies. Two related papers are Fudenberg and Tirole’s (2000) study of consumer poaching and Villas-Boas’ (2004) model of price cycles with overlapping generations of buyers. Chen and Zhang’s environment is a dynamic extension of Varian’s (1980) model of search with some informed (about all posted prices) and some uninformed buyers. In a static version, the model generates price dispersion via mixed strategies. In a dynamic version with two firms, due to randomized pricing, one firm necessarily wins all of the informed consumers in the first period. The firm that *does not* attract any informed consumers discriminates between past and new customers. This sort of targeted pricing relies on heterogeneity in consumers’ search costs rather than their preferences.

Shin and Sudhir (2010) study a repeated version of a Hotelling model where, in each period, some consumers have unit demand, while others have demand for $q > 1$ units. Unlike this paper, buyer types (demands) never change. However, *retailer preference* may change as a fraction of buyers randomly switch to a different location. The market is assumed to be covered, so buyers must purchase their distinguishable demands from some firm. Each firm can target buyers based on whether they purchased 0, 1, or q units earlier. Thus, full separation among *past* customers is assured, while pooling and mixed strategies are ruled out. They derive bounds on demand heterogeneity and address-stochasticity for firms to reward past customers over new ones.

There are several aspects of privacy and dynamic pricing that we do not touch upon. Fudenberg and Villas-Boas (2006) provide a comprehensive overview of behavior-based pricing. For analysis of online and targeted advertising see Evans (2009), Athey and Gans (2010), and Bergemann and Bonatti (2011). In Goldfarb and Tucker (2011), and a series of related contributions, the authors empirically study the impact of regulatory restrictions on the effectiveness of online advertising. Daughety and Reinganum (2010) study the interplay between privacy of individuals’ actions and social perceptions arising from the same. For a computer science perspective on privacy and algorithmic mechanism design, especially on the notion of *differential privacy*, see the survey by Pai and Roth (2013).

3. Economic Environment

A unit measure of buyers make sequential purchase decisions over the products of two monopolistic firms: firm 1 and firm 2. Each buyer has unit demand for each firm’s product. Consumer valuations for each product can be either v_H (high-value buyer) or v_L (low-value buyer) with $v_H > v_L$.⁹ Valuations are private information. Firms are assumed to have zero marginal costs. All buyers and both firms are risk neutral and do not discount the future.

In period 1 each buyer knows her valuation for good 1 but not for good 2. Buyer valuations for the two goods are correlated and follow a two-state Markov process.¹⁰ With probability $\alpha \in [0, 1]$, a buyer’s valuation for good 2 is identical to that for good 1. With probability $(1 - \alpha)$ the valuation switches to the other type. The *persistence* (not transition) probabilities for high and low buyer types are denoted by α_H and α_L , respectively. These probabilities and the *initial fraction* of high-value buyers, denoted by μ_1 , are common knowledge.

In period $t = 1, 2$, firm t makes a take it or leave it offer to every buyer based on the firm’s beliefs about the buyer’s type. We assume that firm 1 does not have any ability to commit to keep a buyer’s purchase decision private. Buyers anticipate disclosure of purchase history, which firm 2 can use to update its beliefs about a buyer’s type. If disclosed information raises firm 2’s expected profit, then firm 1 captures this gain entirely.¹¹

⁹Later, we consider the possibility that the valuations for the two goods have different support, and provide bounds on how large the variations can get, without affecting our results.

¹⁰Other stochastic processes work equally well, provided buyers of a given initial type have similar expectations.

¹¹Lacking commitment, the situation is the same as one with a single firm active in both periods. A single firm internalizes the benefit of joint profit maximization, the allocation of bargaining power between the two firms does

Strategies: A strategy for firm 1 is simply a uniform price $P_1 \in \mathfrak{R}_+$ for all buyers. In each period, a buyer's strategy is a *purchase probability* r based on her valuation, the price offered, and firm 2's beliefs. There are two types of buyers initially, and these can change between periods. The list of buyer strategies based on type realizations is $\{r_H, r_L, r_{HH}, r_{HL}, r_{LH}, r_{LL}\}$. The first subscript refers to period 1 valuation and the second to period 2 valuation. We make the standard assumption that an indifferent buyer purchases with probability one, except when a buyer explicitly uses a mixed strategy.

In period 2 a buyer accepts any offer not exceeding her valuation. Thus, only two prices matter in period 2, $P_2 \in \{v_L, v_H\}$. The strategies σ_2^0 and σ_2^1 represent the probabilities with which firm 2 offers the *low price* conditional on the buyer rejecting, or accepting the offer, respectively. Firm 2 offers v_L if and only if its posterior belief, that a buyer has valuation v_H , is less than v_L/v_H . Firm 2's beliefs are described in the next section. For notational clarity, the dependence of buyers' and firm 2's strategies on prices and beliefs is omitted.

The solution concept is Perfect Bayesian Equilibrium, strategy-belief pairs with the requirement that strategies are Nash given beliefs and other agents' strategies, and beliefs are derived using Bayes' rule. When beliefs are unrestricted by Bayes' rule, wherever possible, we use equilibrium refinement to place restrictions on off-equilibrium beliefs.

4. Equilibrium Analysis

We begin by setting up a reference case without information disclosure. The unconditional posterior probability of a buyer having a valuation v_H is $\mu_2 \equiv \mu_1 \alpha_H + (1 - \mu_1)(1 - \alpha_L)$. In period $t = 1, 2$, firm t offers v_L only if the fraction of high-value buyers in that period is less than the ratio v_L/v_H . Otherwise the firm offers v_H . The expected profit for firm t is then $\max\{v_L, \mu_t v_H\}$. We refer to this as the *Static Outcome*. For purchase history to be valuable, it must raise firm 2's expected profit above this benchmark.

Now consider dynamic pricing. Let $\bar{\alpha} = \max\{\alpha_H, (1 - \alpha_L)\}$ and $\underline{\alpha} = \min\{\alpha_H, (1 - \alpha_L)\}$. Consider two cases: (i) $v_L/v_H \geq \bar{\alpha}$ or (ii) $v_L/v_H \leq \underline{\alpha}$. In either case, even perfect revelation of type in period 1 does not affect firm 2's pricing decision, it always charges v_L in the first case and v_H in the latter. In all interesting cases we must have the following restriction.

Assumption 1: $\underline{\alpha} < v_L/v_H < \bar{\alpha}$.

In period 2, purchase probabilities r_{ij} 's are not really strategic as buyers accept all offers not exceeding their valuation. However, when making their period 1 purchase decision, each buyer anticipates firm 2's optimal reactions. On observing a buyer's period 1 action, firm 2's posterior beliefs (conditional probability of v_H) are given by Bayes' rule as

$$\mu_2^0 = \frac{\mu_1(1 - r_H)\alpha_H + (1 - \mu_1)(1 - r_L)(1 - \alpha_L)}{\mu_1(1 - r_H) + (1 - \mu_1)(1 - r_L)} \quad (1)$$

$$\mu_2^1 = \frac{\mu_1 r_H \alpha_H + (1 - \mu_1) r_L (1 - \alpha_L)}{\mu_1 r_H + (1 - \mu_1) r_L} \quad (2)$$

The superscripts 0 and 1, respectively refer to a buyer's decision to reject or accept firm 1's offer. As previously mentioned, σ_2^0 and σ_2^1 represent the conditional probabilities with which firm 2 offers v_L . The optimal decision for firm 2 is a mapping from posterior beliefs μ_2^i to the relevant σ_2^i , for $i = 0, 1$. Whenever $\mu_2^i > v_L/v_H$, $\sigma_2^i = 0$, and when $\mu_2^i < v_L/v_H$, $\sigma_2^i = 1$. If $\mu_2^i = v_L/v_H$, then firm 2 is indifferent, and may randomize between v_L and v_H .

not matter, and there is no hold-up problem. We use the metaphor of two separate firms to associate fully separating equilibria with valuable consumer data. This is the same approach as in Taylor (2004). In case of targeted advertising, both firms deal with advertising networks serving as information intermediaries.

Define a *continuation equilibrium* as the equilibrium of a game starting after any price announcement by firm 1. In the absence of proper sub-games, this approach allows the use of backward induction.

Lemma 1: *In any continuation equilibrium a high-value buyer must accept any offer that a low-value buyer is willing to accept with positive probability.*

Despite uncertainty in future valuations, this is a standard monotonicity result. Put differently, when purchase by a high-value buyer is uncertain, a low-value buyer must reject the same offer with certainty. Despite the monotonicity, $r_H \geq r_L$ does not imply a clear ordering of the posterior beliefs about a buyer's period 2 valuation. To see this, first consider $r_H = r_L$, which only happens when they are both zero, or both one. In this case, posterior beliefs are either the same as the unconditional value μ_2 , or they are unrestricted by Bayes' rule. Now suppose $r_H > r_L$. There are two contrasting conditions (variations of Assumption 1) which lead to different equilibrium outcomes, allowing for all possible combinations of the two persistence probabilities.

Condition A: $\alpha_H > v_L/v_H > (1 - \alpha_L)$. Environments where the persistence probability of the high-value buyer exceeds the transition probability of the low-value buyer.

Condition B: $\alpha_H < v_L/v_H < (1 - \alpha_L)$, when above likelihoods are reversed.

The two conditions lead to different orderings of μ_2^1 and μ_2^0 . The ordering matters because it is tied to the probability of being offered v_L in period 2. Using (1) and (2), $\mu_2^1 - \mu_2^0 = (r_H - r_L)(\alpha_H + \alpha_L - 1)$. With $r_H > r_L$, we have $\mu_2^1 > \mu_2^0$ only if $\alpha_H + \alpha_L - 1 > 0$, that is, when Condition A holds. In this case, refusal leads to a greater likelihood of a discount than purchase does. This is reversed under Condition B. The hyperplane $\alpha_H + \alpha_L = 1$ separates the two cases. Either condition (A or B) may arise when one type of buyer has valuations that are positively correlated while the other has valuations that are negatively correlated.

For either type of buyer ($i = H, L$) there is a lower bound on period 1 price \underline{P}_i which the buyer must accept with certainty. There also exists an upper bound, \overline{P}_i , (one for each type) such that all offers exceeding this value are rejected with certainty. For a low-value buyer these bounds are $\underline{P}_L = v_L - (1 - \alpha_L)(v_H - v_L)$ and $\overline{P}_L = v_L + (1 - \alpha_L)(v_H - v_L)$. A low-value buyer cannot reject offers lower than \underline{P}_L , or accept offers higher than \overline{P}_L , *even if* doing so guarantees an offer of v_L in period 2. Similarly, we have $\underline{P}_H = v_H - \alpha_H(v_H - v_L)$ and $\overline{P}_H = v_H + \alpha_H(v_H - v_L)$ for high-value buyers. When $\alpha_L < \alpha_H$ we have the following ordering of these thresholds and buyer valuations: $\underline{P}_L < v_L < \underline{P}_H < \overline{P}_L < v_H < \overline{P}_H$.¹²

Lemma 2: *In period 1 all buyers accept offers lower than \underline{P}_L and all buyers reject offers exceeding \overline{P}_H . Low-value buyers reject offers exceeding \overline{P}_L and high-value buyers accept all offers less than \underline{P}_H . Under Condition A, all buyers reject offers exceeding their valuation. Under Condition B, all buyers accept any offer less than their valuation.*

The result arises from a combination of Lemma 1, the construction of the thresholds, and from Conditions A and B. Under Condition A, no buyer ever accepts any period 1 offer that exceeds her valuation. This is a seemingly trivial result. It is only seemingly so because, under Condition B, there exist equilibria in which buyers accept offers exceeding their period 1 valuation. For instance, high-value buyers may do so in anticipation of recouping their losses in the future, something impossible under Condition A. Figure 2 illustrates Lemma 2 by marking various period 1 offers that are accepted or rejected with certainty by different types of buyers (solid lines). It also directs attention to prices where buyer purchase decisions require further analysis (dashed lines).

When firm 2 does not receive any information from firm 1, it offers v_H when $\mu_2 > v_L/v_H$.

¹²If $\alpha_L > \alpha_H$, then \underline{P}_H and \overline{P}_L switch places in the order. \underline{P}_H and \overline{P}_L are never jointly relevant, so this 'sub-order' does not really matter. Condition B can sometimes lead to a negative value for \underline{P}_L . In such cases, the implication is that there may not be any price that a low-value buyer accepts with certainty.

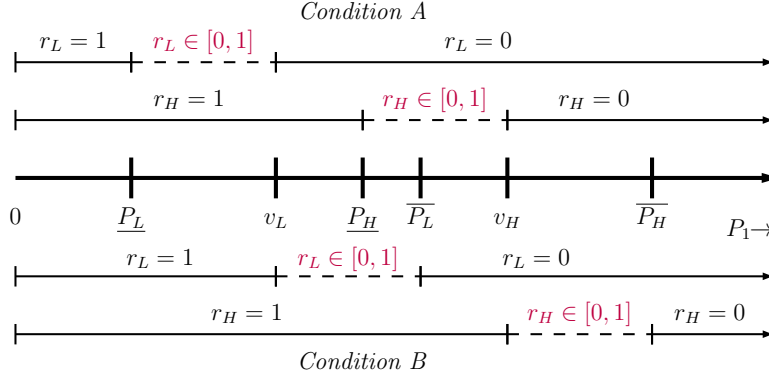


Figure 2: Illustration of Lemma 2 (for $\alpha_L < \alpha_H$)

Fudenberg and Tirole (1983) describe such a firm as a *Tough Seller*. If the unconditional beliefs are such that they lead to the low price, then firm 2 is a *Soft Seller*. These two environments are separated by the surface $v_L/v_H = \mu_1\alpha_H + (1 - \mu_1)(1 - \alpha_L)$, which is obtained by setting the unconditional posterior μ_2 equal to the ratio v_L/v_H .

Tough and Soft Seller conditions reflect firm 2's predisposition without purchase history. With a Tough firm 2, buyer information is valuable because it allows firm 2 to offer *targeted discounts*. With a Soft firm 2, disclosed information is valuable because it allows *targeted markups*.¹³ Conditions A and B help firm 2 decide whom to offer the targeted discounts or markups to. For instance, under Condition A, a *Tough* firm 2 is more likely to offer targeted discounts to those who did not purchase firm 1's product. Using T and S to denote Tough and Soft Seller conditions, we represent the combinations of the two types of environments by: A-T, A-S, B-T, and B-S.

With unit demand, two types of buyers, and uniform price, period 1 outcomes can belong to one of just three classes: Pooling (P), Full Separation (FS), and Partial Separation (PS). Partial separation arises when a buyer is unable to accept or reject offers with certainty. This can take two forms: low-value buyers randomize, while high-value buyers accept with certainty (PS_L), and low-value buyers reject with certainty, while high-value buyers randomize (PS_H). Lemma 1 rules out simultaneous randomization by both types.

Outcomes belonging to FS , PS_H , and PS_L are informative and allow firm 2 to engage in targeted pricing. However, "informativeness" does not necessarily translate to "valuable." As we show later, only FS outcomes lead to valuable purchase history. In other cases, purchase history does not increase firm 2's expected profit. This is the reason why the literature does not find purchase histories to be valuable, without making further assumptions.

Figure 2 suggests that pooling equilibria (class P) are most likely at low price ranges, followed by the classes PS_L , FS , and PS_H (in that order) for increasingly higher period 1 prices. We will see that partial separation can only arise when firm 2 has Tough beliefs. It is useful to view the following results (Propositions 1-4) along with Figure 3, which summarizes buyer behavior in period 1 for each environment.

Proposition 1: *Suppose Condition A-T holds. All buyers accept offers $P_1 \leq P_L$ with certainty (Class P). Offers in $P_L < P_1 \leq P_H$ are rejected by all low-value buyers and accepted by all high-value buyers (Class FS). Price offers in $P_H < P_1 \leq v_H$ are accepted only by a fraction r_H^* of high-value buyers (Class PS_H). In Class P equilibria firm 2 offers v_H to all buyers. In Class FS*

¹³In practice, targeted discounts are easy to implement via coupons. Targeted markups are challenging to implement in some settings, such as online or offline retail, yet relatively easy in others, such as consumer credit and insurance.

equilibria firm 2 offers v_H to all those who accepted firm 1's offer and v_L otherwise. In Class PS_H equilibria firm 2 offers v_H to all buyers who accept firm 1's offer, and buyers who reject firm 1's offer receive an offer of v_L with probability σ_H^* .

$$r_H^* = 1 - \left(\frac{1 - \mu_1}{\mu_1} \right) \left[\frac{v_L/v_H - (1 - \alpha_L)}{\alpha_H - v_L/v_H} \right] \text{ and } \sigma_H^* = \frac{v_H - P_1}{\alpha_H(v_H - v_L)} \quad (3)$$

From Lemma 2 (Figure 2), under Condition A, we know that all buyers accept offers up to \underline{P}_L resulting in Class P equilibria. Firm 2 acts Tough and offers the high price to everyone. Again, from Lemma 2, all high-value buyers accept offers less than \underline{P}_H with certainty. Therefore, we have full separation of types between v_L and \underline{P}_H (Class FS). Using Condition A, firm 2 offers v_L (*targeted discounts*) to those who reject firm 1's offer and v_H to those who accept.

Thus, we are only concerned about two price ranges: $[\underline{P}_L, v_L]$ and $[\underline{P}_H, v_H]$. First, consider prices in $[\underline{P}_L, v_L]$. Ordinarily, both types should accept these offers with certainty. However, low-value buyers strategically reject such offers to *signal* their type. Rejection is costly for a low-value buyer, as she must leave an immediate surplus on the table. However, if rejection is correctly interpreted by firm 2 as a signal of low type, then the buyer is assured of a discount in period 2.¹⁴ This is supported by appealing to Cho and Kreps' (1987) Intuitive Criterion. Rejection increases expected surplus in period 2 as the buyer's valuation may switch to v_H . A high-value buyer can never send the same message. Therefore, prices between \underline{P}_L and v_L also lead to Class FS equilibria.

Next consider prices between \underline{P}_H and v_H . Accepting such offers leads to an immediate surplus of $v_H - P_1$ for the high-value buyers. However, under Condition A, this also guarantees a high price in the future. Moreover, if all high-value buyers accepted the offer, then firm 2 would offer a discount to all those who rejected the offer. This makes rejection very tempting to high-value buyers. Therefore, high-value buyers cannot accept with certainty. However, high-value buyers cannot reject these offers with certainty either. If all buyers reject firm 1's offer, then firm 2 learns nothing and acts Tough. Therefore, high-value buyers must randomize their purchase decision for price offers that lie between \underline{P}_H and v_H .

Intuitively, a higher price reduces a high-value buyer's immediate surplus from purchase and makes rejection attractive. However, it also reduces the probability of receiving a discount in the second period (on rejecting the offer). The expected surplus is $v_H - P_1$ irrespective of whether a buyer accepts or rejects. The *randomized purchase* probability r_H^* is such that it makes firm 2 exactly indifferent between offering v_L or v_H , conditional on observing a *rejection*. That is, r_H^* ensures that $\mu_2^0 = v_L/v_H$, which also explains why it is independent of the price. Firm 2's mixed strategy, however, is derived from the buyer's indifference condition. Thus, σ_H^* depends on the period 1 price. Note that as $P_1 \rightarrow v_H$, firm 2's mixed strategy becomes degenerate as $\sigma_H^* \rightarrow 0$.

Proposition 2: *Suppose Condition A-S holds. Both types of buyers accept offers $P_1 \leq v_L$ with certainty (Class P). Offers in $v_L < P_1 \leq \underline{P}_H$ are rejected by all low-value buyers and accepted by all high-value buyers (Class FS). Offers exceeding \underline{P}_H are rejected by all buyers. In Class P equilibria firm 2 offers v_L to all buyers. In Class FS equilibria firm 2 offers v_H to all those who accepted firm 1's offer and v_L otherwise.*

Consider offers between \underline{P}_L and v_L . From Lemma 2, all high-value buyers accept with certainty. Unlike the case with a Tough Seller, low-value buyers no longer have an incentive to reject these offers. Even if rejection is interpreted as low-type, there is nothing to be gained from it because firm 2 is Soft. Thus, all buyers accept all offers up to v_L leading to Class P equilibria. Firm 2

¹⁴As far as we are aware, Kennan (1998) presents the first instance of such *strategic* rejections. Such rejections cannot occur in environments without uncertainty. These rejections never actually arise in equilibrium as they are pre-empted by firm 1. However, their possibility influences equilibrium outcomes.

acts Soft and offers the low price to everyone. Note that firm 2's beliefs are completely unrestricted following a rejection. Unlike the previous setting with a Tough Seller, belief refinements have no bite in this case as both types of buyers necessarily lose from rejecting the offer. Thus, there are several off-equilibrium beliefs that can sustain the same *unique outcome*.

High-value buyers accept all offers up to \underline{P}_H with certainty. Since low-value buyers reject anything higher than v_L we have fully separating equilibria between v_L and \underline{P}_H . As firm 2 had Soft beliefs, purchase leads to *targeted markups*. Finally, note that all high-value buyers reject offers exceeding \underline{P}_H . If they accept, then using Condition A, firm 2 will offer the high price leaving them with zero surplus. Given that firm 2 is Soft, it is better to pass up on any immediate surplus from purchase in the first period and wait for the guaranteed low price in the future.

Proposition 3: *Suppose Condition B-T holds. Both types of buyers accept offers $P_1 \leq v_L$ with certainty (Class P). Offers in $v_L < P_1 \leq \overline{P}_L$ are accepted by a fraction r_L^* of low-value buyers and all high-value buyers (Class PS_L). Price offers in $\overline{P}_L < P_1 \leq \overline{P}_H$ are rejected by all low-value buyers and accepted by all high-value buyers (Class FS). In Class P equilibria firm 2 offers v_H to all buyers. In Class PS_L equilibria firm 2 offers v_H to all buyers who reject firm 1's offer, buyers who accept firm 1's offer are offered v_L with probability σ_L^* . In Class FS equilibria firm 2 offers v_L to all buyers who accept firm 1's offer and v_H to those who reject.*

$$r_L^* = \left(\frac{\mu_1}{1 - \mu_1} \right) \left[\frac{v_L/v_H - \alpha_H}{(1 - \alpha_L) - v_L/v_H} \right] \text{ and } \sigma_L^* = \frac{P_1 - v_L}{(1 - \alpha_L)(v_H - v_L)} \quad (4)$$

From Lemma 2, we only need to concern ourselves with low-value buyers' strategies in (v_L, \overline{P}_L) , and high-value buyers' strategies in (v_H, \overline{P}_H) . Consider offers in (v_L, \overline{P}_L) first. All high-value buyers must accept with certainty. Note that low-value buyers can neither reject nor accept these offers with certainty. If they reject with certainty, then they are identified, and Condition B guarantees a high price in the future. If low-value buyers accept with certainty, then firm 2 learns nothing and acts Tough. This leaves the low-value buyers with a negative surplus in the first period and zero in the second. Therefore, low-value buyers must randomize. The rest of the result is similar in spirit to Proposition 1. The randomized purchase probability r_L^* is such that, on observing a purchase, firm 2 is exactly indifferent between offering v_L or v_H . Firm 2's offers v_L with probability σ_L^* conditional on purchase in the first period. Firm 2 offers v_H to any buyer who rejects the offer. As in Proposition 1, only firm 2's mixed strategy is affected by price. Within this range, the strategy for either type of buyer is independent of the price.

Now consider prices in the range (v_H, \overline{P}_H) . Ordinarily, all buyers ought to reject such offers with certainty as the price exceeds valuation for either type. However, since firm 2 is Tough, rejection by all buyers guarantees a high price in the future. Under Condition B, high-value buyers have an incentive to reveal their type. Thus, even though accepting such offers is associated with an immediate loss in surplus, high-value buyers may accept an offer as a signal to firm 2. As in Proposition 1, this is supported by appealing to a forward induction on beliefs argument. If this *strategic purchase* is interpreted correctly as a signal, then it assures the high-value buyers a low price in the future. In the second period, firm 2 offers v_L (*targeted discount*) to all those who accept such offers, and v_H to those who do not.

Proposition 4: *Suppose Condition B-S holds. Both types of buyers accept offers $P_1 \leq \overline{P}_L$ with certainty (Class P). Offers in $\overline{P}_L < P_1 \leq v_H$ are rejected by all low-value buyers and accepted by all high-value buyers (Class FS). Offers exceeding v_H are rejected by all buyers. In Class P equilibria firm 2 offers v_L to all buyers. In Class FS equilibria firm 2 offers v_L to all those who accepted firm 1's offer and v_H otherwise.*

As in Proposition 3, we restrict attention to buyer responses to prices in (v_L, \overline{P}_L) , and

(v_H, \overline{P}_H) . Lemma 2 already characterizes buyer strategies for other prices. When $P_1 \in (v_L, \overline{P}_L)$, then low-value buyers must accept with certainty. This is despite the fact that the price is above their valuation v_L . Since firm 2 is Soft, purchase guarantees the low price in the future. From Lemma 2, high-value buyers always accept these offers. Should a low-value buyer reject the offer, then using Condition B, firm 2 offers v_H in the second period. Therefore accepting the offers is a weakly dominant strategy for low-value buyers. Thus we have pooling for prices up to \overline{P}_L , and firm 2 acts Soft.

Lemma 2 (Figure 2) then implies full separation between \overline{P}_L and v_H . This leads firm 2 to offer v_H (targeted markups) to all who reject firm 1's offer, and v_L to those who purchased. Finally, consider prices in (v_H, \overline{P}_H) . Unlike Proposition 3 with a Tough seller, there is no incentive for a high-value buyer to accept these offers to signal their type. Even without any signaling the Soft seller condition guarantees the low price. Thus, all buyers reject offers exceeding v_H . Both types of buyers are hurt by accepting such offers, so equilibrium refinements have no bite. As in Proposition 2 (Condition A-S), equilibrium *outcome* is unique, but there is a multiplicity of off-equilibrium beliefs, on observing a purchase.

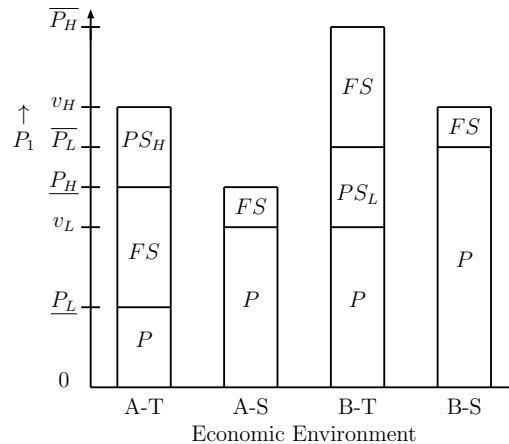


Figure 3: Period 1 outcomes (Propositions 1–4)

Uncertainty and Full Separation

Figure 3 summarizes first-period outcomes derived in Propositions 1–4, and helps explain why uncertainty is key to our results. Consider perfect positive correlation as in Acquisti and Varian (2005), or perfect negative correlation considered in Calzolari and Pavan (2006). In either case, all price ranges marked *FS* in Figure 3 disappear. For instance, when $\alpha_L = \alpha_H = 1$, then $\underline{P}_H = \underline{P}_L = v_L$ and the range of prices for full separation for Conditions A-T or A-S collapses. Similarly, when $\alpha_L = \alpha_H = 0$, then $\overline{P}_H = \overline{P}_L = v_H$, and the range of fully separating prices under Conditions B-T or B-S vanishes. Even with imperfect correlation, full separation cannot arise when buyers know their future valuation in period 1 itself, as they do in Taylor (2004). Without uncertainty, only buyers with valuation v_H in each period have strategic relevance, and there are no thresholds such as \underline{P}_H or \underline{P}_L any more.

Furthermore, without uncertainty in future valuations, there cannot be any signaling via strategic rejections or strategic purchases. Only buyers with valuation v_L in the first period who *know* their future valuation is v_H have an incentive to strategically reject offers less than v_L as a signal. However, should they attempt to do so (without uncertainty), then they are identified and

not offered any discount. Similarly, high-value buyers can no longer signal their type by accepting a price offer higher than v_H (under Condition B).

Value of Consumer Information

Proposition 5: *Consumer purchase history is valuable only under Class FS equilibria. The value of consumer information is: $w_{AT} = (1 - \mu_1)[v_L - (1 - \alpha_L)v_H]$ under Condition A-T, $w_{AS} = \mu_1[\alpha_H v_H - v_L]$ under Condition A-S, $w_{BT} = \mu_1[v_L - \alpha_H v_H]$ under Condition B-T, and $w_{BS} = (1 - \mu_1)[(1 - \alpha_L)v_H - v_L]$ under Condition B-S.*

The value of purchase history is a product of the measure of consumers in the targeted group (for discounts or markups), and the per-consumer gain in expected profit relative to the no-information case. For instance, under Condition A-T, the gain in firm 2's expected profit (from targeted discounts) is $v_L - (1 - \alpha_L)v_H$ for each consumer with the low valuation in the first period, and the fraction of such consumers is $(1 - \mu_1)$. The product of these components is denoted by w_{AT} . The other expressions have similar interpretations.

Partial separation also leads to targeted pricing, but purchase history has no value. Partially separating equilibria only arise when firm 2 has Tough beliefs. For instance, in Class PS_H outcomes (Proposition 1) purchase guarantees v_H in the future, and rejection leads to v_L with probability σ_H^* . When firm 2 offers v_H to those who accepted, it earns the same revenue as the static case. Whenever it randomizes, it expects the same revenue from offering v_H or v_L . Therefore, purchase history is not valuable in partially separating equilibria.

Purchase history loses value as $\mu_1 \rightarrow 0$, or when $\mu_1 \rightarrow 1$ as, in either case, all buyers are nearly identical. Purchase histories also lose value as $v_L/v_H \rightarrow \alpha_H$ or $v_L/v_H \rightarrow (1 - \alpha_L)$, that is, in situations that come close to violating Assumption 1. In such cases the per-consumer gain from targeted pricing drops to zero.

w_{AT} can be written as $(1 - \mu_1)[v_L/v_H - (1 - \alpha_L)]v_H$. For a given μ_1 , this is highest at the boundary separating Tough and Soft Seller environments, that is, when $v_L/v_H = \alpha_H \mu_1 + (1 - \alpha_L)(1 - \mu_1)$, represented by dashed diagonals in Figure 1. At this boundary, $w_{AT} = \mu_1(1 - \mu_1)(\alpha_H + \alpha_L - 1)v_H$. In fact, all expressions for value of purchase data work out to be similar and are given by $\mu_1(1 - \mu_1)|\alpha_H + \alpha_L - 1|v_H$. Thus, consumer data are more valuable for intermediate values of μ_1 and when the probabilities α_H and $(1 - \alpha_L)$ are very different from each other. Of course, this is mostly of academic interest, unless, one also assumes that firms can influence these parameters.

Equilibrium Outcomes

As noted earlier, Figure 3 summarizes the conditions under which various (continuation) equilibrium classes arise. Within a given class, firm 1's expected profit strictly increases as it raises the price. For example, under Condition A-T, any price in $(\underline{P}_L, \underline{P}_H)$ leads to the same buyer strategies, and thus, the same value for information. For all continuation equilibria of this type, firm 1's expected profit is highest at the upper bound \underline{P}_H . Increasing the price to the upper bound of prices supporting *partially* separating equilibria (PS_L or PS_H) makes firm 2's mixed strategies, σ_L^* or σ_H^* , degenerate. That is, for Class PS_H as $P_1 \rightarrow v_H$, using (3), $\sigma_H^* \rightarrow 0$. Similarly, for Class PS_L as $P_1 \rightarrow \overline{P}_L$, using (4), $\sigma_L^* \rightarrow 1$. Of course, buyers' randomizations (r_L^* or r_H^*) are independent of P_1 , which is why nothing of consequence changes. Thus, in any environment, firm 1's optimal decision is to simply compare the expected profit from using the upper bound of every class of equilibria that can arise in that environment.

Let π_E^c denote the maximum value of firm 1's expected profit for class 'c' and environment 'E.' The 'candidate' values for firm 1's optimal expected profit for each environment are: $\pi_{AT}^P = \underline{P}_L$,

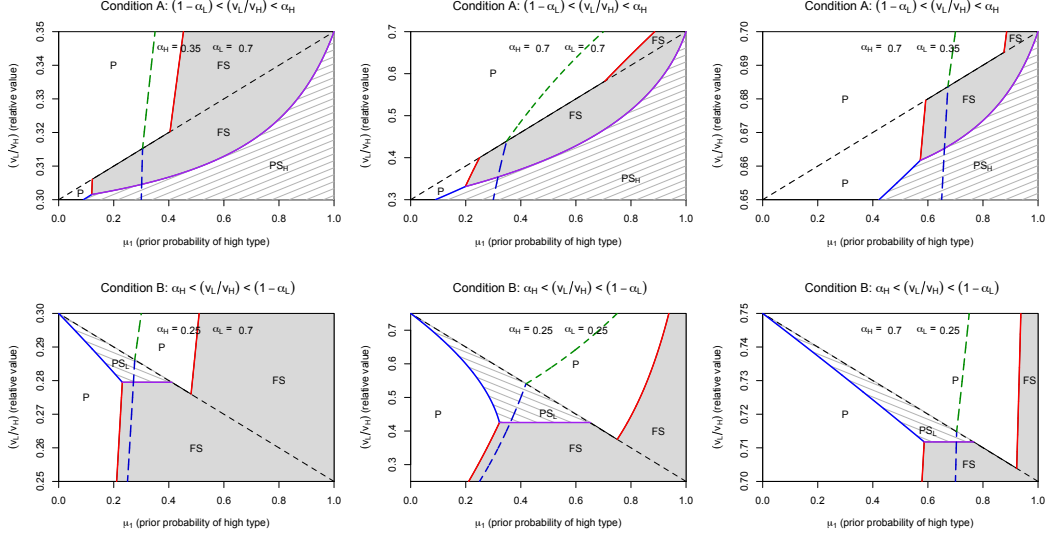


Figure 4: Equilibrium phase transitions

$\pi_{AT}^{FS} = \mu_1 \underline{P}_H + w_{AT}$ and $\pi_{AT}^{PSH} = \mu_1 r_H^* v_H$ (under Condition A-T), $\pi_{AS}^P = v_L$ and $\pi_{AS}^{FS} = \mu_1 \underline{P}_H + w_{AS}$ (under Condition A-S), $\pi_{BT}^P = v_L$, $\pi_{BT}^{FS} = \mu_1 \overline{P}_H + w_{BT}$ and $\pi_{BT}^{PSL} = [\mu_1 + r_L^* (1 - \mu_1)] \overline{P}_L$ (under Condition B-T), and $\pi_{BS}^P = \overline{P}_L$ with $\pi_{BS}^{FS} = \mu_1 v_H + w_{BS}$ (under Condition B-S). For any environment, it is straightforward (though tedious) to verify that there exist parameter values that can make any of these optimal.

Adding up the possibilities for all four environments, equilibrium can take ten potential forms (from Figure 3 or, all the π_E^c 's above). Formal characterization of Perfect Bayesian Equilibrium is a list of optimal strategies and beliefs: P_1 and μ_1 for firm 1, r_L and r_H for the buyers in period 1, firm 2's beliefs: μ_2 , μ_2^0 and μ_2^1 , and strategies: σ_2^0 and σ_2^1 , and lastly, period 2 strategies for the four possible buyer types: r_{LL}, r_{LH}, r_{HL} , and r_{HH} . For brevity, we omit formal statements of equilibrium, all ten types can be stated formally in the given format using Propositions 1-5.

To get a feel for when consumer data are valuable for targeted pricing we plot optimal equilibrium outcomes over the parameter space. In Figure 1, presented earlier, the persistence probabilities α_H and α_L are chosen exogenously. Condition A or B, whichever is relevant, provides endogenous bounds on the ratio v_L/v_H . The separation between the Soft and Tough seller conditions is illustrated via *dashed diagonals* representing the equality between v_L/v_H and the unconditional posterior μ_2 , that is, $v_L/v_H = (1 - \alpha_L) + [\alpha_H + \alpha_L - 1]\mu_1$. This is an upward sloping diagonal under Condition A, and a downward sloping one under Condition B. In either case, the space above the diagonal implies that firm 2 is a Soft Seller. Within Tough or Soft conditions, *solid curves* represent the boundaries separating different classes of equilibrium outcomes. The *dashed curves* represent boundaries between targeted-pricing outcomes when consumers are not aware of the practice, we explain these more fully in the next section where we use them for welfare comparisons.

Figure 4 shows how equilibrium outcomes change with varying persistence levels. As in Figure 1, the *solid grey* regions represent full separation, leading to valuable purchase histories. Regions with *shading lines* represent partial separation, and regions with no shading represent pooling. For reference, the figures in the center are the same as Figure 1. These reference plots show where different equilibrium outcomes arise when buyer valuations display positive serial correlation with

$\alpha_H=\alpha_L=0.7$ (top center), and with negative serial correlation $\alpha_H=\alpha_L=0.25$ (bottom center). The other panels have serial correlation of *mixed* signs: $\alpha_H=.35, \alpha_L=0.7$ (top left), $\alpha_H=.7, \alpha_L=0.35$ (top right), $\alpha_H=.25, \alpha_L=0.7$ (bottom left), and $\alpha_H=.7, \alpha_L=0.25$ (bottom right).

Changes in outcomes across the panels in Figure 4 have simple explanations. In the panels on the left, α_H is closer to 0 and α_L to 1, giving high-value buyers a strong incentive to *reveal* their type while low-value buyers have a weak incentive to do the same. This makes full separation easy, and provides intuition behind the large areas of solid grey. In the panels on the right, the opposite happens as α_H approaches 1 and α_L gets closer to 0. Now, all buyers have a strong incentive to *conceal* their type, which makes full separation difficult and explains why the areas of solid grey shrink.

From Assumption 1, the endogenous values for v_L/v_H (y-axis) range from $\min\{\alpha_H, 1 - \alpha_L\}$ to $\max\{\alpha_H, 1 - \alpha_L\}$. The closer the sum of α_H and α_L gets to one, the narrower the range of interesting ratios becomes. It is unlikely that the sum of persistence probabilities is very close to one in any market of interest. Figure 4 intentionally depicts some relatively extreme situations to illustrate which way the outcomes shift.

Different Support of Valuations

We briefly discuss the possibility that buyer valuations for the two products have different supports. Suppose valuations for the second product can be either $v_L^F \neq v_L$ or $v_H^F \neq v_H$, with $v_H^F > v_L^F$. Low-value buyers rarely earn a surplus in period 1 (only in region II_a in Figure 1), their incentives are driven entirely by changes in future surplus. This depends on the gap in future valuations, $\Delta v^F \equiv v_H^F - v_L^F$, and the probability of type inversion, $(1 - \alpha_L)$.

For high-value buyers, there is an inter-temporal trade-off at play. As the gap in future valuations, Δv^F , becomes smaller compared to the gap in period 1, $\Delta v \equiv v_H - v_L$, their incentive to conceal (or reveal) becomes weaker, and our results of valuable purchase history continue to hold. When Δv^F gets larger relative to Δv , then the high type's incentive to conceal (or reveal) becomes stronger. In this case, our results continue to hold provided the gap in future valuations does not become too large.

It is easy to verify that our monotonicity result (Lemma 1) remains valid as long as Δv^F is no larger than $\min\{\Delta v/\alpha_H, \Delta v/(1 - \alpha_L)\}$. If this condition is violated, then either $\overline{P}_H < v_L$, or $\overline{P}_L > v_H$, and Lemma 1 may not hold. For example, suppose $\alpha_L = 3/4$ and $\alpha_H = 1/4$, then the gap in valuations for the second product should be no larger than four times the gap in the valuations for the first product.¹⁵

5. Welfare Implications

Consider three information disclosure regimes: (1) *Static*: Purchase data are available but, for whatever reason, are not used for targeted pricing, (2) *Opportunistic*: Purchase data are available and used for targeted pricing without the knowledge of consumers (naïve or unaware consumers), and (3) *Dynamic*: Purchase data are used for targeted pricing with full knowledge of consumers.

In the static case, depending on $\max\{v_L, \mu_t v_H\}$, the price in period $t = 1, 2$ is either v_L or v_H . Thus, there are four combinations of static prices. In the opportunistic case, firms engage in dynamic pricing without consumers' knowledge. When there are very few high-value buyers, firm 1 can do no better than simply sell to everyone at v_L , and the outcomes coincide with the static case.

¹⁵An alternative interpretation is that, if the supports of valuations are identical, then consumer data can be sold to up to four downstream firms.

In other cases it offers a price v_H in period 1, which all high-value buyers accept. Under Condition A, firm 2 offers v_H to all those who purchased firm 1's product, others are offered v_L . Under Condition B, the offers are reversed. The value of purchase data is the same as in Proposition 5.

Suppose Condition A-T holds. Firm 1's expected profit from opportunistic pricing is $\mu_1 v_H + w_{AT}$. This leads to a separation of the parameter space with a boundary at $v_L = \mu_1 v_H + w_{AT}$ between opportunistic and static outcomes. All four boundaries are marked by *dashed curves* in Figure 1. Each one has the same form described above $v_L = \mu_1 v_H + w_{ij}$, where the subscript ij refers to combinations of Conditions A or B, and Tough or Soft. Static and opportunistic outcomes are the same for parameter combinations to the left of these boundaries.

Suppose the economy transitions from one information disclosure regime to another. Let ΔCS_L denote the corresponding change in expected surplus of initially low-value buyers. Similarly, ΔCS_H is the change in expected surplus of initially high-value buyers, and $\Delta \pi_1$ represents the change in firm 1's expected profit. Firm 2's expected profit is always the same in every regime. Finally, ΔTS denotes change in social or total surplus. We use *superscripts* to denote which of Condition A or B applies.

Proposition 6: {Static to Opportunistic} *Under Condition A: $\Delta CS_L^A \geq 0$, $\Delta CS_H^A \leq 0$, $\Delta \pi_1^A \geq 0$, and $\Delta TS^A \leq 0$, except when the static prices are high in each period. Under Condition B: $\Delta CS_L^B \leq 0$, $\Delta CS_H^B > 0$ except when firm 2 is Soft, $\Delta \pi_1^B \geq 0$ in all cases, and $\Delta TS^B \geq 0$, except when static prices are low in each period.*

Moving from static to opportunistic regime can *never lower* period 1 price from v_H to v_L . If that were the case, the optimal static price could not have been v_H . Therefore, firm 1's expected profit can never decrease with opportunistic pricing, and no consumer can gain additional surplus in period 1 from the transition to opportunistic disclosure regime. Welfare outcomes depend on who gets the low price in period 2 under opportunistic pricing.

Under Condition A, those who do not accept an offer of v_H in the first period are offered v_L in the second. This is why, under Condition A, high-value buyers are never better off with opportunistic pricing and low-value buyers are never worse off. High-value buyers are strictly worse off in the opportunistic regime when static prices are v_H in the first period and v_L in the second. Low-value buyers are strictly better off when static prices are v_H in each period, which is also when total surplus is highest with opportunistic pricing.

Under Condition B, high-value buyers get a discount in period 2 when they purchase in the first (at v_H). So they are usually better off. The exception is when firm 2 is Soft and static price is low in the second period. Under Condition B, low-value buyers face higher prices with dynamic pricing, thus they are never better off. Therefore, the welfare consequences get partially reversed. The measure of trades is typically higher than the static case, so social welfare also rises, with the exception of situations where static prices are v_L in each period.

Now consider the more interesting transition from an opportunistic regime to a dynamic one, as initially-unaware consumers *become* aware of targeted pricing and respond strategically. As the following result shows, consumers do not always benefit from their awareness of dynamic pricing.

Proposition 7: {Opportunistic to Dynamic} (using Figure 1). *Under Condition A: $\Delta CS_L^A \geq 0$, except in region IV_b , $\Delta CS_H^A \geq 0$, except in regions II_b and II_c , $\Delta \pi_1^A \leq 0$, and $\Delta TS^A \geq 0$, except in regions II_b , II_c , and IV_b . Under Condition B: $\Delta CS_L^B \geq 0$ except in region V , $\Delta CS_H^B \geq 0$ except in regions V , VI_b , VI_c and $VIII_b$, $\Delta \pi_1^B \geq 0$ in all cases, and $\Delta TS^B \geq 0$, except region VI_c , where it is negative, and region VI_b where it can take either sign.*

Under Condition A, strategic rejections, or the possibility thereof, lowers the joint profit of both firms compared to the opportunistic regime. Firm 1 would benefit from committing to full privacy, as its expected profit is higher in the static case (no information disclosure). The problem, of course, is that firm 1 has no ability to commit to nondisclosure.

Under Condition A, low-value buyers have an incentive to reject offers to reveal their type, which is why information disclosure usually never hurts them. The one exception is region IV_b , where strategic rejections by high-value buyers lead to high prices in both periods. This is a negative externality on the low type, as they no longer receive the discounts they do under opportunistic pricing. Due to lower prices, high-value buyers are typically better off than the opportunistic case. The only exceptions are in regions II_b and II_c where dynamic pricing leads firm 1 to raise prices from v_L (in the opportunistic case) to either fully or partially separating ones. In these regions, low-value buyers are unaffected, but total surplus is lower. Total surplus also drops in region IV_b as the level of trade is reduced in both periods, first due to randomized purchase by high-value buyers in period 1, and because firm 2 offers just v_H in period 2 (degenerate mixed strategy).

Under Condition B, buyers' incentives to conceal or reveal their type get reversed. Despite their willingness to be identified, high-value buyers do not always gain from information disclosure. This happens for two reasons. Under Condition B, low-value buyers can conceal their type only by accepting higher prices. This allows firm 1 to raise prices, which is a negative externality imposed by low-value buyers on the high type (regions V and VI_b). In some cases, high-value buyers accept offers exceeding their valuation as a signal (regions VI_c and $VIII_b$). Firm 1 is never worse off as prices are generally higher than the opportunistic case.

In region VI_b the loss of welfare of high-value buyers is sometimes offset by the gain in firm 1's revenue, thus, raising total surplus. This is the only region where economic efficiency may rise or fall with information disclosure. However, it is straightforward to further differentiate this region such that the welfare consequences are unambiguous. The unambiguous loss in efficiency in region VI_c is due to costly signaling by high-value buyers.

6. Conclusion

Empirical implications: We study the value of purchase data when buyers anticipate targeted pricing. We show that purchase data are valuable only when buyers are uncertain about future valuations. The nature of preference uncertainty is such that buyers of a given type (market segment) have similar expectations about how their outside options may evolve. There are simple endogenous bounds on demand-heterogeneity (valuations), based on persistence probabilities for both types of buyers. Within these bounds, purchase data are most valuable when downstream firms have beliefs bordering Tough and Soft conditions. Naturally, for purchase data to be valuable, the initial fraction of high-value buyers must be neither too high, nor too small.

The analysis suggests that buyers may sometimes engage in strategic refusals or strategic purchases. Strategic refusals (or purchases) intended to *conceal* type arise in partially separating equilibria (classes PS_L and PS_H). When overall demand is known, we can test for randomized purchases by comparing acceptance and refusal rates with known type distributions. Strategic rejections (or purchases) intended as signals to *reveal* type can be identified via price experimentation through offers that lie outside *known* support of valuations.

Consumer finance can serve as a potential testing ground for our results, as consumers are well aware of targeted pricing based on credit history. A simple example of building histories of both purchases and *credible refusals* is the use of electronic balance transfer offers for credit cards. Unless a customer responds to a paper-based offer, it is hard to distinguish those who considered it (but did not pursue) from those who did not even open it. However, with electronic offers, banks can keep track of "viewed offers," and gain further insights through *messaging opportunities*, such as customers clicking on "Remind me later," or, "No thanks." Data brokers' ability to combine offline and online behavior can help implement the concept in several markets. In sender-receiver and bargaining games "saying no" matters. Data-brokers and advances in information technology

make it possible for consumers to have their refusals recorded in credible ways.

The theory is relevant for buyers including businesses. The financial crisis of 2008 provides some “potential” (admittedly *speculative*) instances. Early withdrawals of some banks from the U.S. Treasury’s Troubled Assets Relief Program (TARP) may be viewed as strategic rejections (of low offers), signaling expectation of continued access to cheap capital, without government support (positive serial correlation in outside options). Goldman Sachs’ decision to sell warrants to Berkshire Hathaway at a dividend of 10% is indicative of a strategic or premium ‘purchase’ (of capital), suggesting both parties believed Goldman’s options for raising capital were very likely to improve in the future (negative serial correlation in outside options).

Policy implications: In its recent report, FTC (2014), the U.S. Federal Trade Commission makes several policy recommendations regarding the market for consumer data. Chief among them are raising consumer awareness of data brokers’ business practices and improving transparency. For example, just like consumer credit reporting agencies, data brokers could be required to list their sources, include mechanisms for consumers to inspect and correct their records, or possibly opt-out at some levels of information collection or disclosure.

The analysis in this paper, and the literature, suggest that – depending on market microstructure – welfare outcomes of targeted pricing can go either way. In some cases, restrictions may harm both consumers and firms. Thus, the desirability of opt-out mechanisms is currently unclear, and requires empirical analysis. Furthermore, strategic sophistication on the part of consumers is meaningless without the right information input. Our analysis suggests that data brokers should be *incentivized* to disclose aggregate information on the distribution and evolution of preferences. Otherwise, consumers remain at a strategic disadvantage, as businesses will continue to have superior information about the distribution of types and trends in various segments.

Obviously, this is easier said than done. Data brokers will be justifiably hesitant to reveal anything that indirectly discloses proprietary information or algorithms. More importantly, data brokers maintain records on thousands of consumer attributes and data segments, which could be overwhelming for consumers to parse. For the information to be useful, it has to be available in a format that ordinary consumers can query and interact with. This is where advances in tools for data visualization and information retrieval could be beneficial in making the information comprehensible to consumers, small enterprises, and policymakers.

PROOFS

Lemma 1: Suppose $r_L > 0$. This holds when $P_1 \leq v_L + (1 - \alpha_L)(v_H - v_L)(\sigma_2^1 - \sigma_2^0)$. Now assume $r_H < 1$. This holds when $P_1 \geq v_H + \alpha_H(v_H - v_L)(\sigma_2^1 - \sigma_2^0)$. For both of these conditions to hold together we must have $1 \leq (1 - \alpha_L - \alpha_H)(\sigma_2^1 - \sigma_2^0)$. However, both terms in the product belong to $(-1, 1)$ leading to a contradiction. Therefore, if $r_L > 0$ then $r_H = 1$.

Lemma 2: $r_L = 1$ for $P_1 < \underline{P}_L$, even if $\sigma_2^0 = 1$ (from Lemma 1, $r_H = 1$). Similarly: $r_L = 0$ for $P_1 > \overline{P}_L$, $r_H = 1$ for $P_1 < \underline{P}_H$, and $r_H = 0$ for $P_1 > \overline{P}_H$ (from Lemma 1, $r_L = 0$). When Condition A holds, $\sigma_2^1 < \sigma_2^0$. Now suppose $r_L > 0$ for some $P_1 > v_L$. This requires $v_L - P_1 \geq (1 - \alpha_L)(v_H - v_L)(\sigma_2^1 - \sigma_2^0)$. Since the term on the left is negative this contradicts Condition A. Thus, under Condition A, $r_L = 0$ for $P_1 > v_L$. Similarly, under Condition A, $r_H = 0$ for $P_1 > v_H$. Using a similar argument, under Condition B, $r_L = 1$ for $P_1 < v_L$ and $r_H = 1$ for $P_1 < v_H$.

Proposition 1: Suppose Condition A-T holds. Consider $P_1 \in (\underline{P}_H, v_H)$. From Lemma 2, $r_L = 0$. Suppose $r_H = 1$. Then Condition A implies $\sigma_2^1 = 0$ and $\sigma_2^0 = 1$. This provides an incentive to a high-value buyer to deviate and reject. Therefore, $r_H \neq 1$. Now suppose $r_H = 0$. Then firm 2 acts Tough, $\sigma_2^0 = 0$. Thus, $r_H \neq 0$ as well. The expected payoff from accepting is $[v_H - P_1]$ and the expected payoff from rejecting is $\alpha_H(v_H - v_L)\sigma_2^0$. Setting these to be equal (indifferent buyer) we get σ_H^* in (3). For firm 2 to randomize, μ_2^0 must equal v_L/v_H . This leads to r_H^* in (3).

Now consider $P_1 \in (\underline{P}_L, v_L)$. From Lemma 2, $r_H = 1$. Suppose $r_L = 1$. Since firm 2 is Tough, purchase leads to an immediate surplus of $v_L - P_1$, and zero surplus in period 2. However, firm 2's beliefs are undefined following a rejection. In this case, we require firm 2 to have reasonable beliefs in the sense of Cho and Kreps (1987). A high-value buyer can never reject such an offer. Therefore, firm 2 must draw the correct inference that the buyer's type was low in period 1. For a low-value buyer, Condition A guarantees a low price in the second period with an expected payoff of $(1 - \alpha_L)(v_H - v_L)$. This exceeds the payoff from accepting. Therefore, $r_L = 0$ for all $P_1 \in (\underline{P}_L, v_L]$. **Proposition 2:** Suppose Condition A-S holds. Consider $P_1 \in (\underline{P}_L, v_L)$. From Lemma 2 $r_H = 1$. If $r_L = 1$, then firm 2 offers v_L to everyone. Suppose a low-value buyer rejects this offer, perhaps to signal their type. By rejecting the offer, low-value buyers give up an immediate surplus and do not gain any additional surplus. So all buyers are strictly better off from accepting. If a rejection is nevertheless observed, then firm 2 is free to believe anything. Irrespective of firm 2's beliefs, it is weakly dominant strategy for low-value buyers to accept. Therefore, $r_L = 1$. Next, consider $P_1 \in (\underline{P}_H, v_H)$. From Lemma 2, $r_L = 0$ and rejection leads to a low price for everyone ($\sigma_2^0 = 1$). Accepting such offers is associated with an immediate surplus of $v_H - P_1$ and zero surplus in the future ($\sigma_2^1 = 0$). But this is less than expected surplus of $\alpha_H(v_H - v_L)$ obtained from rejecting. Thus, it is a weakly dominant strategy for all high-value buyers to reject such offers.

Proposition 3: Suppose Condition B-T holds. Consider $P_1 \in (v_L, \overline{P}_L)$. From Lemma 2, $r_H = 1$. If $r_L = 0$, then Condition B implies $\sigma_2^0 = 0$. Therefore, $r_L \neq 0$. Now suppose $r_L = 1$. Then $\mu_2^1 = \mu_2$ and Condition T implies $\sigma_2^1 = 0$. Thus $r_L \neq 1$. Using arguments similar to Proposition 1, we can derive the mixed strategies r_L^* and σ_L^* in (4). Next consider $P_1 \in (v_H, \overline{P}_H)$. From Lemma 2, $r_L = 0$. Suppose $r_H = 0$. Then, Condition T implies $\sigma_2^0 = 0$. Note that μ_2^1 is indeterminate. As in Proposition 1, we appeal to belief refinements and require firm 2 to interpret purchase as a signal of high (period 1) valuation. In this case, $\mu_2^1 = \alpha_H$, which by Condition B leads to $\sigma_2^1 = 1$. Purchase leads to positive expected surplus and rejection leads to zero surplus. Therefore, $r_H = 1$.

Proposition 4: Suppose Condition B-S holds. Consider $P_1 \in (v_L, \overline{P}_L)$. From Lemma 2, $r_H = 1$. Suppose $r_L < 1$. Then $\mu_2^0 = 1 - \alpha_L$, and Condition B implies $\sigma_2^0 = 0$. When $r_L = 1$, then $\mu_2^1 = \mu_2$ and $\sigma_2^1 = 1$ due to Condition S. Thus, $r_L = 1$. Next consider $P_1 \in (v_H, \overline{P}_H)$. From Lemma 2, $r_L = 0$. Suppose, as in Proposition 3, $r_H > 0$ as a signal, then $\mu_2^1 = \alpha_H$, and Condition B guarantees $\sigma_2^1 = 1$. However, due to Condition S, a low price is guaranteed even from rejection, $\mu_2^1 = \mu_2$ and $\sigma_2^1 = 1$. As in Proposition 2, belief refinements have no bite in the Soft seller setting and equilibrium is not unique. But the equilibrium outcome is unique.

Proposition 5: (i) Suppose Condition A-T holds. The static (no information) expected profit to firm 2 is $\pi_{AT}^{static} = \mu_1 \alpha_H v_H + (1 - \mu_1)(1 - \alpha_L)v_H$. Under Class *FS* equilibria, the expected profit is $\pi_{AT}^{FS} = \mu_1 \alpha_H v_H + (1 - \mu_1)v_L$. The value of purchase history is the difference $w_{AT} = \pi_{AT}^{FS} - \pi_{AT}^{static} = (1 - \mu_1)[v_L - (1 - \alpha_L)v_H]$. Similarly, for the remaining environments we have: (ii) $\pi_{AS}^{static} = v_L$, $\pi_{AS}^{FS} = \mu_1 \alpha_H v_H + (1 - \mu_1)v_L$, and $w_{AS} = \mu_1[\alpha_H v_H - v_L]$. (iii) $\pi_{BT}^{static} = \mu_1 \alpha_H v_H + (1 - \mu_1)(1 - \alpha_L)v_H$, $\pi_{BT}^{FS} = \mu_1 v_L + (1 - \mu_1)(1 - \alpha_L)v_H$, and $w_{BT} = \mu_1(v_L - \alpha_H v_H)$. (iv) $\pi_{BS}^{static} = v_L$, $\pi_{BS}^{FS} = \mu_1 v_L + (1 - \mu_1)(1 - \alpha_L)v_H$, and $w_{BS} = (1 - \mu_1)[(1 - \alpha_L)v_H - v_L]$. In Class *PS_H* equilibria (Condition A-T), firm 2 expects to earn $\mu_1 \alpha_H r_H^* v_H + [\mu_1 \alpha_H (1 - r_H^*) + (1 - \mu_1)(1 - \alpha_L)]v_H = \mu_2 v_H$. This is the same as the no-information case. Similarly, in Class *PS_L* equilibria (Condition B-T), the expected profit is the same as the static case ($\sigma_2^0 = 0$, and $\sigma_2^1 = \sigma_L^*$). Thus, purchase history has no value.

Proposition 6 and **Proposition 7** are straightforward to verify using results previously described. These appear in an appendix.

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Appendix (Omitted Proofs)

Proof of Proposition 6: Refer to Figure 5, which separates static outcomes into high or low prices in period 1 based on whether μ_1 exceeds v_L/v_H , or not. Static and opportunistic outcomes are the same in regions 1, 2, 5, and 6. Thus there are no changes in welfare for any agent in these regions.

In regions 3a, 3b, 4a, and 4b, using Condition A, opportunistic prices are $P_1 = v_H, \sigma_2^0 = 1$ and $\sigma_2^1 = 0$. *Region 3a:* Static prices are $P_1 = P_2 = v_L$. CS_L is $(1 - \alpha_L)(v_H - v_L)$ in each case. Thus, $\Delta CS_L = 0$. CS_H drops from $(1 + \alpha_H)(v_H - v_L)$ to 0, thus, $\Delta CS_H < 0$. π_1 increases from v_L to $\mu_1 v_H + w_{AS}$, thus, $\Delta \pi_1 > 0$. The measure of trades drops from 2 to $\mu_1 + \alpha_H \mu_1 + (1 - \mu_1)$. Thus, $\Delta TS < 0$. *Region 3b:* Static prices are $P_1 = v_H, P_2 = v_L$. $\Delta CS_L = 0$ as in region 3a. CS_H drops from $(\alpha_H)(v_H - v_L)$ to 0, thus, $\Delta CS_H < 0$. π_1 increases from $\mu_1 v_H$ to $\mu_1 v_H + w_{AS}$, thus, $\Delta \pi_1 > 0$. The measure of trades drops from $1 + \mu_1$ to $\mu_1 + \alpha_H \mu_1 + (1 - \mu_1)$. Thus $\Delta TS < 0$. *Region 4a:* Static prices are $P_1 = v_L, P_2 = v_H$. CS_L increases from 0 to $(1 - \alpha_L)(v_H - v_L)$, thus, $\Delta CS_L > 0$. CS_H drops from $(v_H - v_L)$ to 0, thus, $\Delta CS_H < 0$. π_1 increases from v_L to $\mu_1 v_H + w_{AT}$, thus, $\Delta \pi_1 > 0$. The measure of trades drops from $1 + \alpha_H \mu_1 + (1 - \alpha_L)(1 - \mu_1)$ to $\mu_1 + \alpha_H \mu_1 + (1 - \mu_1)$. Thus $\Delta TS < 0$. *Region 4b:* Static prices are $P_1 = P_2 = v_H$. $\Delta CS_L > 0$, as in region 4a. CS_H is 0 in both cases, thus, $\Delta CS_H = 0$. π_1 increases from $\mu_1 v_H$ to $\mu_1 v_H + w_{AT}$, thus, $\Delta \pi_1 > 0$. The measure of trades increases from $\mu_1 + \alpha_H \mu_1 + (1 - \mu_1)(1 - \alpha_L)$ to $\mu_1 + \alpha_H \mu_1 + (1 - \mu_1)$. Thus $\Delta TS > 0$.

In regions 7a, 7b, 8a, and 8b, using Condition B, opportunistic prices are $P_1 = v_H, \sigma_2^0 = 0$ and $\sigma_2^1 = 1$. Thus, $CS_L = 0$, $CS_H = \alpha_H(v_H - v_L)$, $\pi_1 = \mu_1 v_H + w_{BS}$ (in regions 7a and 7b), and $\pi_1 = \mu_1 v_H + w_{BT}$ (in regions 8a and 8b). The measure of trades is $2\mu_1 + (1 - \mu_1)(1 - \alpha_L)$. *Region 7a:* Static prices are $P_1 = P_2 = v_L$. $CS_L = (1 - \alpha_L)(v_H - v_L)$, thus, $\Delta CS_L < 0$ on moving to opportunistic regime. CS_H is $(1 + \alpha_H)(v_H - v_L)$, thus $\Delta CS_H < 0$. In the static case π_1 is v_L . Thus, $\Delta \pi_1 > 0$. The measure of trades is 2. Thus, $\Delta TS < 0$. *Region 7b:* Static prices are $P_1 = v_H, P_2 = v_L$. $\Delta CS_L < 0$, as in region 7a. CS_H is $(\alpha_H)(v_H - v_L)$, thus, $\Delta CS_H = 0$. In the static case π_1 is $\mu_1 v_H$. Thus, $\Delta \pi_1 > 0$. The measure of trades is $1 + \mu_1$. Thus $\Delta TS > 0$. *Region 8a:* Static prices are $P_1 = v_L, P_2 = v_H$. $CS_L = 0$ in the static case. Thus, $\Delta CS_L = 0$ on moving to opportunistic regime. CS_H is $(v_H - v_L)$, thus $\Delta CS_H > 0$ on moving to the opportunistic regime. In the static case π_1 is v_L . Thus, $\Delta \pi_1 > 0$. The measure of trades is $1 + \alpha_H \mu_1 + (1 - \alpha_L)(1 - \mu_1)$. As consumer and producer surplus are both higher, $\Delta TS > 0$. *Region 8b:* Static prices are $P_1 = P_2 = v_H$. $CS_L = CS_H = 0$ in the static case. Thus, $\Delta CS_L = 0$ on moving to opportunistic regime and $\Delta CS_H > 0$. In the static case π_1 is $\mu_1 v_H$. Thus, $\Delta \pi_1 > 0$. The measure of trades is $\mu_1 + \alpha_H \mu_1 + (1 - \alpha_L)(1 - \mu_1)$. Thus, $\Delta TS > 0$.

Proof of Proposition 7: Refer to Figure 1. *Region I:* Opportunistic and dynamic prices are the same, $P_1 = P_2 = v_L$. Thus, there is no change in surplus for any agent.

In regions II_a, II_b and II_c , opportunistic prices are $P_1 = v_L$ and $P_2 = v_H$. The benchmark expected values for consumer and firm 1's surplus are $CS_L = 0, CS_H = v_H - v_L$ and $\pi_1 = v_L$. The measure of trades is $1 + \mu_2$. *Region II_a:* Dynamic pricing strategies are: $P_1 = \underline{P}_L, \sigma_2^0 = 1, \sigma_2^1 = 1$. Since $\underline{P}_L < v_L$, $\Delta CS_L > 0$, $\Delta CS_H > 0$, $\Delta \pi_1 < 0$. As there is no change in the measure of trades $\Delta TS = 0$. *Region II_b:* Dynamic pricing strategies are: $P_1 = v_H, \sigma_2^0 = \sigma_H^* = 0, \sigma_2^1 = 0$. Low-value buyers never earned any surplus to start with, so $\Delta CS_L = 0$. High-value buyers no longer earn any surplus, thus $\Delta CS_H = -\alpha_H(v_H - v_L) < 0$. Firm 1 earns $\mu_1 r_H^* v_H$ instead of v_L . Given that the opportunistic price was v_L rather than v_H , this implies $\Delta \pi_1 < 0$. Obviously, $\Delta TS < 0$. *Region II_c:* Dynamic pricing strategies are: $P_1 = \underline{P}_H, \sigma_2^0 = 1, \sigma_2^1 = 0$. Low-value buyers expect a surplus of $(1 - \alpha_L)(v_H - v_L) \Rightarrow \Delta CS_L > 0$. High-value buyers earn no surplus. So, $\Delta CS_H < 0$. Firm 1's expected profit drops from v_L to $\mu_1 \underline{P}_H + w_{AT}$, thus $\Delta \pi_1 < 0$. The measure of trades drops from

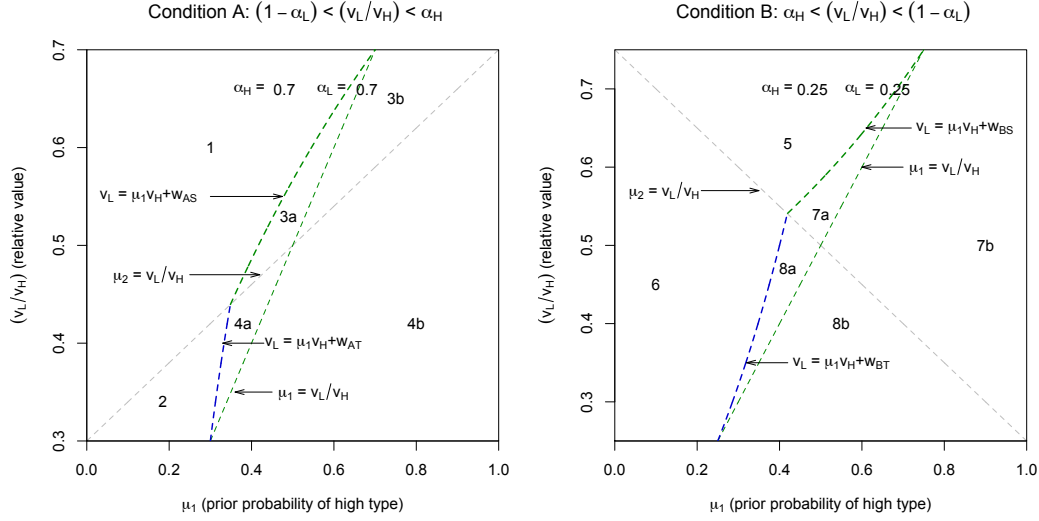


Figure 5: Welfare Comparison (Static to Opportunistic)

$1 + \mu_2$ to $\mu_1 + \mu_1\alpha_H + (1 - \mu_1)$. Thus, $\Delta TS < 0$.

In regions III_a , III_b , IV_a and IV_b opportunistic prices are $P_1 = v_H$, $\sigma_2^0 = 1$, $\sigma_2^1 = 0$. The reference level for consumer and producer surplus are: $CS_L = (1 - \alpha_L)(v_H - v_L)$, $CS_H = 0$, $\pi_1 = \mu_1 v_H + w_{AS}$ in regions III_a , III_b , and $\pi_1 = \mu_1 v_H + w_{AT}$ in regions IV_a and IV_b . The reference level of trade is $\mu_1 + \mu_1\alpha_H + (1 - \mu_1)$. *Region III_a* : Dynamic pricing strategies are: $P_1 = v_L, P_2 = v_L$. $CS_L = (1 - \alpha_L)(v_H - v_L)$. Thus $\Delta CS_L = 0$. $CS_H = (v_H - v_L)(1 + \alpha_H)$. Thus $\Delta CS_H > 0$. $\pi_1 = v_L$, thus $\Delta\pi_1 < 0$. The measure of trades is 2, and $\Delta TS > 0$. *Region III_b* : Dynamic pricing strategies are: $P_1 = \underline{P}_H$, $\sigma_2^0 = 1$, $\sigma_2^1 = 0$. Low-value buyers earn the same expected surplus. Thus, $\Delta CS_L = 0$. High-value buyers earn an expected profit of $v_H - \underline{P}_H$. Thus $\Delta CS_H > 0$. Firm 1 sells to the same measure of consumers at a lower price, and the value of consumer information is identical. Thus, $\Delta\pi_1 < 0$. The measure of trades is identical in each case: $\mu_1 + \mu_1\alpha_H + (1 - \mu_1) \Rightarrow \Delta TS = 0$. *Region IV_a* : Dynamic pricing strategies are: $P_1 = \underline{P}_H$, $\sigma_2^0 = 1$, $\sigma_2^1 = 0$. Same outcomes as III_b . *Region IV_b* : Dynamic pricing strategies are: $P_1 = v_H$, $\sigma_2^0 = 0$, $\sigma_2^1 = \sigma_H^* = 0$. Since $\sigma_2^0 = 0$, $\Delta CS_L < 0$. High-value buyers have surplus in both cases, thus $\Delta CS_H = 0$. Firm 1's profit drops from $\mu_1 v_H + w_{AT}$ to $r_H^* \mu_1 v_H$, $\Rightarrow \Delta\pi_1 < 0$. The measure of trades drops from $\mu_1 + \mu_1\alpha_H + (1 - \mu_1)$ to $r_H^* \mu_1 + \mu_2 \Rightarrow \Delta TS < 0$.

Region V: Opportunistic prices are $P_1 = v_L$ and $P_2 = v_L$. In the dynamic case, $P_1 = \overline{P}_L$, $\sigma_2^0 = 1$, $\sigma_2^1 = 1$. Low-value buyers earn an expected surplus of $(1 - \alpha_L)(v_H - v_L)$ in the opportunistic case and zero in the dynamic case. Thus, $\Delta CS_L < 0$. High-value buyers earn a surplus of $(v_H - v_L)(1 + \alpha_H)$ in the opportunistic case and a lower value $v_H - \overline{P}_L + \alpha_H(v_H - v_L)$ in the dynamic one. Thus, $\Delta CS_H < 0$. Firm 1's expected profit is v_L in the opportunistic case and \overline{P}_L in the dynamic one. Thus $\Delta\pi_1 > 0$. The measure of trades remains at 2 since all buyers purchase in each period. Thus, $\Delta TS = 0$.

In regions VI_a , VI_b , and VI_c , opportunistic prices are $P_1 = v_L$ and $P_2 = v_H$. The reference levels are $CS_L = 0$, $CS_H = (v_H - v_L)$, $\pi_1 = v_L$, and the measure of trades is $1 + \mu_2$. *Region VI_a* : Dynamic pricing strategies are: $P_1 = v_L$, $\sigma_2^0 = 0$, $\sigma_2^1 = 0$. Since the outcomes are identical there is no change in surplus for any agent. *Region VI_b* : Dynamic pricing strategies are: $P_1 = \overline{P}_L$, $\sigma_2^0 = 0$, $\sigma_2^1 = 1$. $CS_L = 0$ here as well. Therefore, $\Delta CS_L = 0$. High-value buyers earn a surplus of $v_H - \overline{P}_L + \alpha_H(v_H - v_L) = (\alpha_L + \alpha_H)(v_H - v_L)$. From Condition B, this is less than the opportunistic

case. Thus, $\Delta CS_H < 0$. Firm 1 earns \overline{P}_L in the dynamic case versus v_L in the opportunistic case. So, $\Delta\pi_1 > 0$. The measure of trades is $2[\mu_1 + r_L^*(1 - \mu_1)] + (1 - r_L^*)(1 - \mu_1)(1 - \alpha_L)$. The net impact on total surplus is ambiguous. *Region VI_c*: Dynamic pricing strategies are: $P_1 = \overline{P}_H$, $\sigma_2^0 = 0$, $\sigma_2^1 = 1$. $CS_L = 0$. Thus, $\Delta CS_L = 0$. High-value buyers earn zero in expectation. Thus, $\Delta CS_H < 0$. Firm 1 expects to earn $\mu_1 \overline{P}_H + w_{BT}$. Since we are in this region, the expected profit must be higher than the opportunistic price v_L (identical to region VI_a). Therefore $\Delta\pi_1 > 0$. The measure of trades is $2\mu_1 + (1 - \alpha_L)(1 - \mu_1)$. For $\Delta TS \geq 0$ we need $2\mu_1 \geq 1 + \mu_1\alpha_H$ or $\mu_1 \geq 1/(2 - \alpha_H)$. In this region $\mu_1 < v_L/v_H$. Together these conditions require $v_L/v_H \leq \alpha_H$ which violates Condition B. Thus $\Delta TS < 0$.

In regions VII_a, VII_b, VIII_a, and VIII_b opportunistic prices are $P_1 = v_H$, and from Condition B, $\sigma_2^0 = 0$, $\sigma_2^1 = 1$. The reference level for consumer and producer surplus are: $CS_L = 0$, $CS_H = \alpha_H(v_H - v_L)$, $\pi_1 = \mu_1 v_H + w_{BS}$ in regions VII_a, VII_b, and $\pi_1 = \mu_1 v_H + w_{BT}$ in regions VIII_a and VIII_b. The reference level of trade is $2\mu_1 + (1 - \mu_1)(1 - \alpha_L)$. *Region VII_a*: Dynamic pricing strategies are: $P_1 = \overline{P}_L$, $\sigma_2^0 = 1$, $\sigma_2^1 = 1$. Low-value buyers earn zero expected surplus in each case. Thus, $\Delta CS_L = 0$. High-value buyers expect to earn a surplus of $v_H - \overline{P}_L + \alpha_H(v_H - v_L)$. Thus, $\Delta CS_H > 0$. Firm 1's expected profit is \overline{P}_L . Since we are in region VII_a and not region VII_b, $\overline{P}_L > \mu_1 v_H + w_{BS}$. Thus, $\Delta\pi_1 > 0$. The measure of trades is 2 in the dynamic case. Thus, $\Delta TS > 0$. *Region VII_b*: Dynamic pricing strategies are: $P_1 = v_H$, $\sigma_2^0 = 0$, $\sigma_2^1 = 1$. The two cases are identical and there is no change in surplus for any agent. *Region VIII_a*: Dynamic pricing strategies are: $P_1 = \overline{P}_L$, $\sigma_2^0 = 1$, $\sigma_2^1 = 1$. Low-value buyers have zero expected surplus in either case. Thus, $\Delta CS_L = 0$. High-value buyers gain $v_H - \overline{P}_L$ in additional expected surplus and $\Delta CS_H > 0$. Firm 1's expected profit is higher. Since we are in region VIII_a and not VIII_b, the expected profit must be higher than $\mu_1 \overline{P}_H + w_{BT}$. That means it must be higher than the reference value as well. Therefore, $\Delta\pi_1 > 0$. The measure of trades increases from $\mu_1 + \mu_1\alpha_H + (1 - \mu_1)(1 - \alpha_L)$ to $\mu_1 + r_L^*(1 - \mu_1) + \mu_1 + (1 - \mu_1) \Rightarrow \Delta TS > 0$. *Region VIII_b*: Dynamic pricing strategies are: $P_1 = \overline{P}_H$, $\sigma_2^0 = 0$, $\sigma_2^1 = 1$. There is no change for low-value buyers, and $\Delta CS_L = 0$. High-value buyers have zero expected surplus. Thus, $\Delta CS_H < 0$. Firm 1 earns $\mu_1 \overline{P}_H + w_{BT}$. Thus $\Delta\pi_1 > 0$. The measure of trades is identical in both cases. Thus, $\Delta TS = 0$.