



End-to-End Performance Analysis of MIMO-OFDM based Wireless Transceivers in the Presence of Hardware Impairments

By

Parag Aggarwal

PhD-14107

under the guidance of

Dr. Vivek Ashok Bohara

Associate Professor, IIIT-Delhi

submitted

**in partial fulfillment of the requirements for the degree of Doctor of
Philosophy**

to

Indraprastha Institute of Information Technology, Delhi

May, 2019

To my loving parents and sisters . . .

Certificate

This is to certify that the thesis titled **End-to-End Performance Analysis of MIMO-OFDM based Wireless Transceivers in the Presence of Hardware Impairments** being submitted by **Parag Aggarwal** to the Indraprastha Institute of Information Technology-Delhi, for the award of the degree of **Doctor of Philosophy**, is an original research work carried out by him under my supervision. In my opinion, the thesis has reached the standards fulfilling the requirements of the regulations relating to the degree.

The results contained in this thesis have not been submitted in part or full to any other university or institute for the award of any degree/diploma.

May 2019

Dr. Vivek Ashok Bohara

Indraprastha Institute of Information Technology-Delhi
New Delhi-110020

Acknowledgements

“It is good to have an end to journey toward; but it is the journey that matters in the end”

–Ursula K. Le Guin

My Ph.D. journey comprises of a lot of learning and achievements which I culminate with this dissertation. The credit of its success is the people who always stood by me and encouraged me one way or another during the entire journey. I am very grateful to have them in my life. I want to express my gratitude to all of them.

First of all, I would like to thank my Maa, Papa and my sisters, Anshika and Shelka, without them this journey would not have been possible. I will forever remain indebted to them for always supporting me and making countless sacrifices for me in general. I dedicate this dissertation to my family. I would also like to thank my grandmother whose cheerful nature always lifts my spirits. They could not understand what I did or the significance of my research, but they never doubted my ability.

I would like to express my sincere gratitude to my advisor Associate Prof. Vivek Ashok Bohara for his invaluable guidance, continuous encouragement, and constant support which were essential for the completion of this dissertation. I appreciate his enthusiasm, insights and productive suggestions on this research. I admire many of his qualities and seek to learn from him. The way he keeps raising the research standards and devotes himself to whatever he works on will continue to inspire me all along my life. I owe him for encouraging me during my research work and carefully listening to and curating my ideas which allowed me to grow tremendously as a researcher. His immense patience, in-depth knowledge, and advice have had a positive influence on not only my research but also on my further career.

I would also like to thank Prof. Anand Srivastava and Associate Prof. Mohammad S. Hashmi for serving as my committee members and taking out their valuable time for their insightful feedback and useful suggestions on my work. A special mention to Prof. Aditya

Trivedi, my master's advisor, who has been the stepping stone of my research career. His timely guidance enlightened me, and I decided to pursue Ph.D.

Thanks to Neha, Mansi, Hamza, and the alumni of Wirocomm research group, especially Dr. Naveen Gupta for all kinds of discussion and support for my research work. I will miss our semester party a lot. My Ph.D. journey would not have been possible without my best buddies Ambuj and Dhananjay who continuously supported me through thick and thin. Needless to mention the *Awesome Wednesdays* and *Fabulous Fridays* when we partied like crazy. The bond that I have developed with them will continue to enrich my life in all the years to come. I also want to thank all my colleagues Lokender, Akshay, Rahul, Protim, Snigdha, Garvita, for their help, support and a lot of fun we had during my Ph.D. span. A singular mention to Shalabh, my best friend, who is like a second family to me and I can confide in him without getting judged.

Last but not the least, I would also like to thank Admin Staff, especially Mrs. Priti Patel, who have resolved all admin related matters without any delay and made my life so easy here that I could focus only on my research.

Abstract

Wireless communication systems based on multi-antenna multi-carrier modulation techniques such as multi-input-multi-output (MIMO) orthogonal frequency division multiplexing (OFDM) have become the state-of-art method for a broadband data transfer. However, it is still not enough as a standalone to satisfy the ever increasing demand for high data rate transmission. To fulfill this demand, new features such as carrier aggregation (CA) and advanced multi-antenna techniques have been proposed by 3rd Generation Partnership Project (3GPP) in Long Term Evolution-Advanced (LTE-A). On exploiting the advantages of OFDM with CA and advanced multi-antenna techniques, the efficiency of the cellular networks can be significantly improved. However, OFDM signal suffers from high peak-to-average-power ratio (PAPR) problem, and high power amplifiers (HPAs) induce nonlinear distortions to such a high PAPR signal. Furthermore, another hardware impairment, i.e., phase noise caused by non-ideal oscillators significantly degrades the efficiency and performance of the system. These impairments severely limit the gains provided by the OFDM-based systems as compared to the linear systems. Hence, it is imperative to take into account the impact of nonlinear HPA and non-ideal oscillator while evaluating the performance of the OFDM-based systems. As a consequence, this dissertation is mainly focused on analyzing the performance of OFDM based CA and multi-user (MU) MIMO systems in the presence of hardware impairments.

The first part of this dissertation analyzes the impact of nonlinear HPA on multi-band CA-MIMO-OFDM system. The nonlinear behavior of the HPA is modeled by a multi-band generalized memory polynomial (MB-GMP) model. It is shown that the received symbol after down-conversion can be canonically decomposed into complex attenuation factor and additive nonlinear noise. The generalized mathematical expressions of complex attenuation factor and variance of nonlinear noise for any number of aggregated bands with any nonlinearity order and any memory depth of HPA are derived. From the derived expressions, an analytical methodology is proposed to obtain the received signal-to-distortion-plus-noise ratio (SDNR), symbol error rate (SER) and error vector magnitude (EVM) of the nonlinear multi-band CA-

MIMO-OFDM system. The proposed work also provides valuable insights on the impact of number of aggregated carriers on the error performance of a nonlinear CA-MIMO-OFDM system.

In the second part, the performance of CA dual-band (DB) MU-MIMO-OFDM system in the presence of nonlinear HPA is investigated. The HPA nonlinearity is modeled by a two-dimensional GMP model. A transmit preprocessing technique is employed to mitigate the effects of inter-user interference. The generalized expression for a signal-to-interference-plus-noise ratio (SINR) for nonlinear DB MU-MIMO-OFDM system is derived and further utilized to obtain an analytical framework to evaluate the performance of the aforementioned system in terms of SER and average capacity. Furthermore, it is shown that the nonlinear interference is a function of inter-modulation and cross-modulation products from every user and increases as the number of users increases, thus significantly deteriorating the performance of the DB MU-MIMO-OFDM system.

A theoretical framework to study the joint impact of non-ideal oscillators and nonlinear HPAs on the downlink MU-MIMO-OFDM system is proposed in the next part of this dissertation. It is shown that the received data symbol affected by phase noise and HPA nonlinearity can be canonically decomposed into following: 1) the desired signal multiplied by an attenuation factor and common phase error (CPE), 2) inter-channel interference (ICI), and 3) additive nonlinear distortion noise which is uncorrelated with the desired signal. The mathematical expression of an instantaneous signal-to-distortion-plus-interference-plus-noise ratio (SDINR) for MU-MIMO-OFDM system impaired with phase noise and nonlinear HPA is derived and then used to obtain the closed-form expression for both SER and overall capacity.

In the end, this dissertation presents an end-to-end analytical framework to evaluate the performance of the cascaded digital predistorter (DPD) and HPA structure on the MIMO-OFDM system. A joint nonlinear polynomial (JNP) model is proposed to represent the characteristics of the cascaded DPD+HPA architecture. It is shown that the received symbol after the MIMO-OFDM demodulation consists of residual noise and a multiplicative factor. By modeling the residual noise as a complex Gaussian process, an analytical methodology is adopted to obtain the received SDNR of a MIMO-OFDM system in the presence of both DPD and HPA. Further by utilizing the expression of SDNR, a closed-form expression of SER is also derived for a multi-path Rayleigh fading channel. It is shown that the SER of the MIMO-OFDM system in the presence of cascaded DPD+HPA architecture significantly improves and approaches to a linear MIMO-OFDM system.

The proposed frameworks in this dissertation can easily be utilized in various wireless standards and will be useful for a communication engineer to design a link budget for MIMO-OFDM based wireless systems without performing extensive simulations or tedious experiments.

Table of contents

Abstract	viii
List of figures	xii
List of tables	xv
Nomenclature	xvi
1 Introduction	1
1.1 Motivation	1
1.2 Objectives	2
1.3 Major Contributions	3
1.4 Other Contributions	5
1.5 Outline	6
2 Background and Related Work	8
2.1 Preliminaries	8
2.1.1 OFDM	8
2.1.2 Carrier Aggregation	10
2.1.3 Multi-User MIMO	10
2.1.4 Hardware Impairments	12
2.1.4.1 High Power Amplifier	12
2.1.4.2 Phase Noise	14
2.1.5 Predistortion	14
2.2 Related Work	15

3	Multi-Band Carrier Aggregated Nonlinear MIMO-OFDM System	20
3.1	Introduction	20
3.2	System Model	22
3.2.1	CA-OFDM Signal	22
3.2.2	Behavioral Model for HPA	23
3.2.3	MIMO Channel	24
3.3	CA-MIMO-OFDM System	24
3.4	SER Analysis	28
3.5	EVM Analysis	31
3.6	Simulation Results	32
3.6.1	SDR Simulation	33
3.6.2	CA-MIMO-OFDM Simulation	35
3.6.3	EVM Analysis	37
3.7	Conclusion	38
4	MU-MIMO-OFDM System with Nonlinear HPAs	39
4.1	Introduction	39
4.2	System Model	41
4.3	DB MU-MIMO-OFDM System with Linear HPAs	43
4.4	DB MU-MIMO-OFDM System with Nonlinear HPAs	46
4.5	End-to-End Performance Analysis	50
4.5.1	Linear dual-band MU-MIMO-OFDM system	50
4.5.2	Nonlinear dual-band MU-MIMO-OFDM system	53
4.6	Simulation Results and Discussion	54
4.6.1	Nonlinear Interference	55
4.6.2	SER Performance	56
4.6.3	Capacity Analysis	58
4.7	Conclusion	60
5	Joint Impact of Nonlinear HPA and Non-ideal Oscillators	62
5.1	Introduction	62
5.2	System Model	63
5.2.1	Linear MU-MIMO-OFDM System	64
5.2.2	HPA Model	66
5.2.3	Phase Noise Model	67
5.3	MU-MIMO-OFDM System with Nonlinear HPAs and Phase Noise	68
5.4	System Performance Analysis	70

5.4.1	SER Performance	73
5.4.2	Capacity Performance	76
5.5	Simulation Results and Discussion	77
5.5.1	Error Performance	78
5.5.2	Capacity Performance	80
5.6	Conclusion	82
6	Performance of Cascaded DPD+HPA Architecture	84
6.1	Introduction	84
6.2	System Model	85
6.2.1	OFDM Modulator	86
6.2.2	Digital Predistortor	87
6.2.3	High Power Amplifier	88
6.3	Cascaded DPD+HPA Architecture	88
6.4	Received Signal Characterization	89
6.5	Simulation Results	94
6.5.1	SISO-OFDM System	94
6.5.2	MIMO-OFDM System	98
6.6	Conclusion	98
7	Conclusion and Future Work	100
7.1	Summary of the Contributions	101
7.2	Future Work	103
Appendix A	Decomposition of the received complex data symbol	104
Appendix B	Mean of nonlinear noise	106
Appendix C	Variance of nonlinear noise	108
Appendix D	Statistical properties of precoded nonlinear distortion vector	111
Appendix E	Mean and variance of $\zeta(0)$	113
Appendix F	Mean and variance of $\zeta(k' - k)$	115
Appendix G	Proof of <i>Lemma 3</i>	116
Appendix H	The nonlinear coefficients of (6.7)	117

Table of contents	xi
-------------------	-----------

Appendix I Author's Publications	119
--	------------

References	121
-------------------	------------

List of figures

2.1	Different types of Carrier Aggregation	10
2.2	Schematic of Downlink MU-MIMO System	11
2.3	Input-output characteristics of a linear and nonlinear HPA.	12
2.4	A characteristics of DPD and HPA at the transmitter.	15
3.1	CA-OFDM signal with aggregation of B number of CCs.	22
3.2	Distortion due to MB nonlinear transmitter.	23
3.3	Schematic of CA-MIMO-OFDM system.	24
3.4	Analytical (solid lines) and simulation (marker points) results of the complex attenuation factor and variance of nonlinear noise for each subcarrier of different CA-OFDM signals for 16 QAM.	33
3.5	Analytical (solid lines) and simulation (marker points) results of average SER for each subcarrier of different CA-OFDM signals without AWGN for 16 QAM and IBO=3dB.	34
3.6	Analytical (solid lines) and simulation (marker points) results of average SER of different CA-MIMO-OFDM system in Rayleigh fading channel for 16-QAM, $N=2048$	35
3.7	Analytical (solid lines) and simulation (marker points) results of average SER for different diversity gain of CA-MIMO-OFDM system in Rayleigh fading channel for 16-QAM, $N=2048$	36
3.8	Analytical (solid lines) and simulation (marker points) results of average EVM for 2x2 CA-MIMO-OFDM system in Rayleigh fading channel for 16-QAM, $N=2048$	37
4.1	System Architecture of dual-band MU-MIMO-OFDM system.	41

4.2	Solid lines and dotted lines represent the variance of nonlinear interference $\phi(k)$ at each subcarrier of SB and DB MU-MIMO-OFDM system, respectively, with $\text{IBO} = 5\text{dB}$	55
4.3	Performance comparison in terms of average SER of dual-band MU-MIMO-OFDM system and SU-MIMO-OFDM system for different IBOs.	56
4.4	Performance comparison in terms of average SER of two different dual-band MU-MISO-OFDM systems for different IBOs.	57
4.5	Performance comparison in terms of average SER of different MIMO-OFDM systems with $\text{IBO} = 5\text{dB}$	58
4.6	Dotted lines and solid lines represent overall system capacity with linear and nonlinear HPA, respectively. The results are plotted with respect to U and different values of total transmit power P_T for $\text{IBO}=5\text{dB}$	59
4.7	Overall capacity of the nonlinear SB (lower curve) and DB (upper curve) MU-MIMO-OFDM system with different number of users and at different IBOs with $P_T = 15\text{dB}$	60
5.1	System Architecture of Downlink MU-MIMO-OFDM system.	64
5.2	An average SER performance comparison between $(6,[2,2,2])$ MU-MIMO-OFDM system (solid lines) and $(2,2)$ SU-MIMO-OFDM system (dashed lines) for different phase noise.	78
5.3	An average SER performance comparison between $(3,[1,1,1])$ system (solid lines) and $(3,[1,1])$ system (dashed lines) for different phase noise with and without HPA nonlinearity.	79
5.4	Solid lines and dashed lines represent an average SER performance of the MU-MIMO-OFDM system with and without nonlinear HPA, respectively, versus the variance of phase noise.	80
5.5	Theoretical (lines) and simulation (markers) results of overall capacity of MU-MIMO-OFDM system with (solid lines) and without (dashed lines) nonlinear HPA for different total power constraint.	81
5.6	Overall capacity of MU-MIMO-OFDM system with (lower curve) and without (upper curve) nonlinear HPA with different number of users and different variance of phase noise for different total power constraint.	82
6.1	A schematic of the nonlinear MIMO-OFDM system in the presence of DPD.	86
6.2	A characteristics of DPD and HPA at the transmitter.	87
6.3	SDR performance of the transmit signal with different transmitter configuration.	95

6.4	Analytical (solid lines) and simulation (marker points) results of average SER of SISO-OFDM system in the AWGN channel for 16 QAM.	96
6.5	Analytical (solid lines) and simulation (marker points) results of average SER of SISO-OFDM system for different M-QAM.	97
6.6	Analytical (solid lines) and simulation (marker points) results of average SER of different MIMO-OFDM systems for 16 QAM.	97

List of tables

2.1	Comparison between PAPR reduction techniques	9
3.1	Transmit signal EVM (%) for 16-QAM CA-OFDM signal with N=2048. . . .	37
4.1	Overall capacity (bits/sec/Hz) of different CA MU-MIMO-OFDM system with $P_T = 15dB$ and IBO=5dB.	59
5.1	Value of C_u and C'_u for different MU-MIMO-ODFM systems	76
5.2	Value of the different parameters	77
6.1	EVM (%) of the transmit signal in the presence of nonlinear HPA only.	95
6.2	EVM (%) of the transmit signal in the presence of both DPD and HPA.	95

Nomenclature

Notations

$(\cdot)^*$	Conjugate Operator
$\mathcal{CN}(m, \sigma^2)$	Complex Gaussian Distribution with m mean and σ^2 variance.
$E[\cdot]$	Expectation Operator
$Ei[\cdot]$	Exponential Integral
$\text{erfc}(\cdot)$	Error Function
$\mathbf{X}$	Frequency Domain Representation of Matrix
$\mathbf{x}$	Frequency Domain Representation of Vector
$\Gamma(\cdot)$	Gamma Function
$(\cdot)^H$	Hermitian Operator
$\ln(\cdot)$	Natural Logarithm Function
$Q(\cdot)$	Q Function
$(\cdot)^T$	Transpose Operator
$(\cdot)^{-1}$	Inverse Operator
$(\cdot)!$	Factorial Operator
$\bar{\mathbf{X}}$	Time Domain Representation of Matrix
$\bar{\mathbf{x}}$	Time Domain Representation of Vector

$U(\cdot, \cdot, \cdot)$	Confluent Hypergeometric Function
$\text{var}[\cdot]$	Variance of Random Variable

Abbreviations

2D	Two Dimensional
3GPP	Third Group Partnership Project
4G	Fourth Generation
5G	Fifth Generation
ACPR	Adjacent Channel Power Ratio
AM/AM	Amplitude-to-Amplitude Modulation
AM/PM	Amplitude-to-Phase Modulation
AWGN	Additive White Gaussian Noise
BER	Bit Error Rate
BS	Base Station
CA	Carrier Aggregation
CC	Component Carrier
CLT	Central Limit Theorem
CPE	Common Phase Error
CSI	Channel State Information
DB	Dual-Band
DFT	Discrete Fourier Transform
DPD	Digital Predistorter
eCA	Enhanced Carrier Aggregation
EH	Enhanced Hammerstein
ET	Envelope Tracking

EVM	Error Vector Magnitude
FFT	Fast Fourier Transform
GMP	Generalized Memory Polynomial
HPA	High Power Amplifier
i.i.d.	Independent and Identically Distributed
IBO	Input Back-Off
ICI	Inter-Channel Interference
IDFT	Inverse Discrete Fourier Transform
ISI	Inter-Symbol Interference
JNP	Joint Nonlinear Polynomial
LOS	Line-of-Sight
LTE-A	LTE-Advanced
LTE	Long Term Evolution
MB	Multi-Band
MCA	Massive Carrier Aggregation
MIMO	Multi-Input-Multi-Output
MP	Memory Polynomial
MRC	maximal Ratio Combining
MSP	Multiple Signaling and Probabilistic
MUI	Multi-User Interference
MU	Multi-User
NMSE	Normalized Mean Square Error
NR	New Radio
OFDM	Orthogonal Frequency Division Multiplexing

PAPR	Peak-to-Average-Power Ratio
PDF	Probability Density Function
QAM	Quadrature Amplitude Modulation
QoS	Quality-of-Service
QPSK	Quadrature Phase Shift Keying
SB	Single-Band
SDINR	Signal-to-Distortion-plus-Interference-plus-Noise Ratio
SDNR	Signal-to-Distortion-plus-Noise Ratio
SDR	Signal-to-Distortion Ratio
SER	Symbol Error rate
SINR	Signal-to-Interference-plus-Noise Ratio
SNR	Signal-to-Noise Ratio
SSPA	Solid-State Power Amplifier
SU	Single-User
SVD	Single Value Decomposition
TWTA	Traveling Wave Tube Amplifier
WiMAX	Worldwide Interoperability for Microwave Access
WLAN	Wireless Local Area Network
WSSUS	Wide Sense Stationary Uncorrelated Scattering
ZF	Zero-Forcing

Introduction

In this chapter, the motivation and objectives of this dissertation are briefly explained followed by the outline of this dissertation.

1.1 Motivation

Next-generation cellular communication must support high data rate services in order to accommodate the needs of users' applications such as high-definition video streaming, mobile TV, online gaming, internet-of-things [1, 2]. As a consequence, there is an upsurge in data traffic over the wireless networks which necessitates the efficient utilization of available resources. To satisfy these demands, each generation of wireless standards proposes to improve quality-of-service (QoS) and network capacity. One of the suggested solution is using multi-carrier modulated multi-input-multi-output (MIMO) system. Most widely adopted multi-carrier modulation scheme is orthogonal frequency division multiplexing (OFDM) due to its resilient behavior against multi-path delay spread and frequency selective fading channels [3]. It utilizes the available spectrum efficiently and supports high data rate communication. On the other hand, MIMO systems are used to enhance the system throughput, transmission reliability, and network capacity [4]. By exploiting the benefits of MIMO and OFDM together, the performance of wireless communication systems can be improved significantly. However, MIMO-OFDM systems are also inadequate to fulfill the high data rate transmission requirements.

In order to achieve high throughput, fourth generation (4G) long term evolution advanced (LTE-A) introduces some additional attributes such as carrier aggregation (CA), advanced MIMO techniques, device-to-device communication, heterogeneous networks, etc [5]. CA allows scalable widening of effective bandwidth by combining the multiple carriers to support wideband signals [6, 7]. It also provides better resource utilization, spectral efficiency and

increased flexibility in terms of load sharing among carriers and frequency allocation. On the other hand, advanced MIMO technique such as massive MIMO has attracted considerable attention of many wireless researchers due to its high spectral and energy efficiency, and ability to improve the processing complexity of next-generation cellular systems. A multi-user (MU) MIMO system, which is a subset of massive MIMO, can simultaneously serve multiple users by spatially sharing the channel in contrast to point-to-point MIMO system [8]. By sharing the same time-frequency resource among multiple users, MU-MIMO can attain capacity and multiplexing gains without the expansion of bandwidth. On joint application of CA and MU-MIMO techniques, the performance of OFDM based system can be improved significantly.

OFDM has a high dynamic range in their signal envelopes, i.e., high peak-to-average-power ratio (PAPR), due to which it is susceptible to nonlinear distortion in the presence of high power amplifier (HPA) at the transmitter [9]. Extensive research has shown that the performance of the conventional OFDM system is considerably deteriorated in the presence of nonlinear HPA [10–13]. In recent studies, it has also been shown that the nonlinear distortion becomes more severe in the presence of CA techniques. Furthermore, non-ideal oscillators, present at the transmitter and receiver, cause phase noise which introduces inter-channel interference (ICI) in the OFDM system. Thus, the hardware impairments such as nonlinear distortion and phase noise not only compromise the reliability but also restrict the capacity of the wireless communication system. Hence, it is imperative to take into account the impact of hardware impairments while evaluating the performance of the wireless network. This dissertation investigates the performance of CA MU-MIMO-OFDM system in the presence of hardware impairments. It also presents the theoretical framework to analyze the MIMO-OFDM based system in the presence of both digital predistorter (DPD) and nonlinear HPA.

1.2 Objectives

The main aim of this dissertation is to develop an analytical framework to evaluate the end-to-end performance of the MIMO-OFDM based system in the presence of nonlinear transceivers. Specifically, the objectives of this work are as follows:

- To study the impact of nonlinear HPA characterized by different behavioral models on the performance of MIMO-OFDM system.
- To theoretically evaluate the performance of CA-MIMO-OFDM system impaired with nonlinear HPAs.
- To design the CA MU-MIMO-OFDM system and analyze its reliability and capacity in the presence of nonlinear distortion caused by HPAs.

- To investigate the joint impact of nonlinear distortion and phase noise on the performance of MU-MIMO-OFDM system.
- To propose a mathematical representation of the cascaded DPD+HPA architecture in order to analyze the end-to-end performance of MIMO-OFDM system in presence of both DPD and HPA.

1.3 Major Contributions

The major contributions of this dissertation are summarized below along with the relevant publications.

- The impact of nonlinear HPA on multi-band CA-MIMO-OFDM system is analyzed. It is shown that the received symbol after down-conversion can be canonically decomposed into complex attenuation factor and additive nonlinear noise. Their generalized mathematical expressions for any number of aggregated bands with any nonlinearity order and any memory depth of HPA are derived. From the derived expressions, an analytical methodology is proposed to obtain symbol error rate (SER) and error vector magnitude (EVM) of the nonlinear multi-band CA-MIMO-OFDM system. It also provides valuable insights on the impact of number of aggregated carriers on the error performance of a nonlinear CA-MIMO-OFDM system.
 - **P. Aggarwal** and V. A. Bohara, “On the Multi-Band Carrier Aggregated Nonlinear LTE-A System,” *IEEE Access*, vol. 5, pp. 16930-16943, 2017.
 - **P. Aggarwal**, A. Agarwal, V. A. Bohara, “Error Vector Magnitude Analysis for Carrier Aggregated OFDM signals with Nonlinear HPA,” 2016 IEEE International Conference on Advanced Networks and Telecommunications Systems (ANTS), Bangalore, 2016, pp. 1-3.
 - **P. Aggarwal**, V. A. Bohara, “Characterization of HPA Using Two Dimensional General Memory Polynomial for Dual Band Carrier Aggregated MIMO-OFDM Systems,” 2016 IEEE International Conference on Communications (ICC), Kuala Lumpur, 2016, pp. 1-7.
 - P. Singhal, **P. Aggarwal**, V. A. Bohara, “Nonlinear distortion analysis of multi-band carrier aggregated OFDM signals,” 2015 IEEE International Conference on Advanced Networks and Telecommunications Systems (ANTS), Kolkata, 2015, pp. 1-6.

- P. Singhal, **P. Aggarwal**, V. A. Bohara, “Analysis of carrier aggregated OFDM signals in presence of dual band power amplifiers,” 2015 Twenty First National Conference on Communications (NCC), Mumbai, 2015, pp. 1-6.
- The performance of CA dual-band (DB) MU-MIMO-OFDM system in the presence of nonlinear HPA is investigated. A transmit preprocessing technique is employed to mitigate the effects of inter-user interference. Using the generalized expression of signal-to-interference-plus-noise ratio (SINR), the analytical expression of SER and average capacity for nonlinear DB MU-MIMO-OFDM system are derived. It is shown that the HPA nonlinearity introduces an irreducible nonlinear interference which is a function of inter-modulation and cross-modulation products from every user. The nonlinear interference increases as the number of users increases, thus significantly deteriorating the performance of the DB MU-MIMO-OFDM system.
 - **P. Aggarwal** and V. A. Bohara, “Analytical Characterization of Dual-Band Multi-User MIMO-OFDM System with Nonlinear Transmitter Constraints,” IEEE Transactions on Communications, vol. 66, no. 10, pp. 4536-4549, Oct 2018.
 - **P. Aggarwal** and V. A. Bohara, “A Nonlinear Downlink Multiuser MIMO-OFDM Systems,” IEEE Wireless Communications Letters, vol. 6, no. 3, pp. 414-417, June 2017.
 - **P. Aggarwal**, F. Jabin and V. A. Bohara, “Nonlinear Amplification Effects on Dual Band Multi-User MIMO-OFDM Systems,” 2018 IEEE International Conference on Communications (ICC), Kansas City, MO, USA, 2018, pp. 1-6.
- A theoretical framework is proposed to study the joint impact of non-ideal oscillators and nonlinear HPAs on the downlink MU-MIMO-OFDM system. It is shown that the received data symbol affected by phase noise and HPA nonlinearity can be canonically decomposed into following parts, namely 1) the desired signal multiplied by an attenuation factor and common phase error (CPE) 2) ICI and 3) additive nonlinear distortion noise which is uncorrelated with the desired signal. On utilizing the mathematical expression of an instantaneous signal-to-distortion-plus-interference-plus-noise ratio (SDINR), the closed-form expressions for both SER and overall capacity for MU-MIMO-OFDM system impaired with phase noise and nonlinear HPA are derived.
 - **P. Aggarwal**, A. Pradhan, V. A. Bohara, “A Downlink Multi-User MIMO-OFDM System with Non-Ideal Oscillators and Amplifiers: Characterization and Performance Analysis,” IEEE Transactions on Wireless Communications, 2018. (Submitted)

- Proposed a joint nonlinear polynomial (JNP) model to represent the characteristics of the cascaded DPD+HPA architecture. It is shown that the received symbol after the MIMO-OFDM demodulation consists of residual noise and a multiplicative factor. The residual noise depends on the nonlinearity order of HPA and can be reduced by increasing the nonlinearity order of DPD. The multiplicative factor depends on the gain of the HPA and DPD. Hence, by modeling the residual noise as a complex Gaussian process, an analytical methodology is adopted to obtain the closed-form expression of SER for the nonlinear MIMO-OFDM system in the presence of the DPD. It is shown that the SER of the MIMO-OFDM system in the presence of cascaded DPD+HPA architecture significantly improves and approaches to a linear MIMO-OFDM system.
- **P. Aggarwal** and V. A. Bohara, “End-to-End Theoretical Evaluation of a Nonlinear MIMO-OFDM System in the Presence of Digital Predistorter,” *IEEE Systems Journal*, pp. 1-11, 2018.

1.4 Other Contributions

Apart from the main contributions as mentioned above, the author of this thesis has also contributed to the following publications:

- **P. Aggarwal**, T. Kabra, R. Ahmad, V. A. Bohara, A. Srivastava, “Adaptive Learning Architecture based Predistorter for Nonlinear VLC system,” in *Photonic Network Communications*, 2019. (Accepted).
- M. H. N. Shaikh, A. Agarwal, **P. Aggarwal**, V. A. Bohara, “On the Spectral Content of Nonlinear Carrier Aggregated Windowed OFDM Systems,” 2018 IEEE International Conference on Advanced Networks and Telecommunications Systems (ANTS), Indore, 2018, pp. 1-5.
- **P. Aggarwal**, A. Agarwal, V. A. Bohara, “On the Spectral Content of the Nonlinearly Amplified Carrier Aggregated OFDM System,” 2018 IEEE Wireless Communications and Networking Conference (WCNC), Barcelona, Spain, 2018, pp. 1-5.
- **P. Aggarwal**, R. Ahmad, V. A. Bohara, A. Srivastava, “Adaptive Predistortion Technique for nonlinear LED with Dimming Control in VLC system,” 2017 IEEE International Conference on Advanced Networks and Telecommunications Systems (ANTS), Bhubaneswar, India, 2017, pp. 1-6.

These works are not explained in this dissertation; however, they are relevant to this dissertation as two of them evaluate the end-to-end performance of nonlinear visible light communication and other provides the insights on the spectral content of the CA-OFDM signals with closed-form expression of their power spectral density.

1.5 Outline

This dissertation is organized as follows:

Chapter 2 presents the background and related work of this dissertation. Initially, the necessary background knowledge about the OFDM with its pros and cons is briefly explained. This chapter also provides the concept of CA and MU-MIMO techniques. It is followed by a discussion on the HPA nonlinearity and different HPA behavioral models. The literature related to the characterization of nonlinear HPAs and its impact on the performance of the OFDM based wireless communication systems is also discussed in this chapter. Further, the performance limitation of the OFDM systems in the presence of phase noise is also reviewed.

Based on the discussion in Chapter 2, the impact of nonlinear HPAs on the multi-band CA-MIMO-OFDM system is analyzed in Chapter 3. The closed-form generalized expressions of nonlinear components such as complex attenuation factor and variance of additive Gaussian nonlinear noise are derived that are applicable to any number of aggregated bands with any nonlinearity order and memory depth. The combined effect of number of aggregated bands, input back-off (IBO) and diversity gain on the performance of CA-MIMO-OFDM system in terms of SER is also investigated.

Both CA and MU-MIMO techniques are features of next-generation wireless standards. In Chapter 4, an analytical framework to evaluate the end-to-end performance of a DB MU-MIMO-OFDM system in the presence of transmitter nonlinearity is proposed. The closed-form expressions of SER and network capacity of the nonlinear DB MU-MIMO-OFDM system in a multi-path Rayleigh fading channel are derived. Simulation results are also presented to validate the efficacy of the proposed analysis.

Chapter 3 and 4 investigate the performance of MIMO-OFDM based systems in the presence of nonlinear HPA only. However, a non-ideal oscillator has not been considered which also deteriorates the efficiency of wireless communication systems. Chapter 5 studies the impact of non-ideal oscillators along with nonlinear HPAs on the performance of MU-MIMO-OFDM system. The joint impact of non-ideal oscillators and nonlinear HPAs can be quantified in terms of multiplicative and additive nonlinear distortions as well as phase noise which causes ICI. Further, comparisons are drawn between the performance of an ideal MU-MIMO-OFDM system and MU-MIMO-OFDM system in the presence of above impairments.

In general, the linearization techniques such as DPD are used to mitigate the nonlinear distortion caused by HPA. Although their performances are quantified in terms of adjacent channel power ratio (ACPR) and EVM at transmitter, the end-to-end performance of wireless communication system in the presence of both DPD and HPA are generally overlooked. In Chapter 6, we theoretically evaluate the end-to-end performance of a MIMO-OFDM system impaired by nonlinear HPAs in the presence of DPD. To represent the behavior of the cascaded DPD+HPA architecture, the JNP model is proposed. By utilizing the JNP model, we can quantify the residual distortion in terms of error performance of the MIMO-OFDM system.

Chapter 7 concludes the dissertation and infers the possible future research directions.

Background and Related Work

In this chapter, the first part provides a brief overview of OFDM, CA, MU-MIMO, hardware impairments and DPD. The second part of this chapter reviews the performance of the aforementioned techniques in the presence of hardware constraints.

2.1 Preliminaries

2.1.1 OFDM

Wireless communication systems based on multi-carrier modulation have become the state-of-art method of broadband data transfer. OFDM is the most well-known and widely deployed scheme among them. Its favorable features such as signal generation using fast Fourier transform (FFT) algorithms, robustness against inter-symbol interference (ISI) and multi-path fading, and easy application of MIMO techniques differentiate it from the rest [3, 4]. OFDM converts high-speed serial data stream into low speed parallel data streams, also known as “subcarriers” that are orthogonal to each other. This property alleviates the impact of frequency selective fading due to multi-path. These sub-carriers are then modulated by a conventional modulation scheme such as quadrature phase shift keying (QPSK), quadrature amplitude modulation (QAM), etc., thus maintaining data rates similar to conventional single-carrier modulation schemes in the same bandwidth. The main advantage of OFDM over single-carrier system is its resilience to frequency selective fading thus eliminating the use of expensive and complicated time domain equalizers which are commonly used in single-carrier systems. Moreover, OFDM is resilient to inter-symbol interference due to the presence of cyclic prefix before every symbol. Owing to these advantages, OFDM and its variants have been widely used in various wireless technologies such as IEEE 802.11 wireless local area

Table 2.1: Comparison between PAPR reduction techniques

PAPR Reduction Technique	BER Increment	Data Rate Loss	Implementation Complexity	Power Increase
Clipping & Filtering	Yes	No	Low	No
Companding	Yes	No	Low	No
Selective Mapping (SLM)	No	Yes	High	No
Partial Transmit Sequence (PTS)	No	Yes	High	No
Interleaving	No	Yes	High	No
Tone Injection (TI)	No	No	High	Yes
Tone Reservation (TR)	No	Yes	High	Yes
Active Constellation Extension (ACE)	No	No	High	Yes
Constrained Constellation Shaping (CCS)	No	No	High	No
Linear Block Coding (LBC)	No	Yes	Low	No
Golay Complementary Sequences (GCS)	No	Yes	High	No

network (WLAN), IEEE 802.16e worldwide interoperability for microwave access (WiMAX), 3rd group partnership project (3GPP) LTE and LTE-Advanced [4].

OFDM consists of large number of independent subcarriers which means that the resultant signal has significant PAPR. A high PAPR signal is susceptible to the nonlinear distortion caused due to the presence of HPA at the transceivers. As a consequence, the performance of the OFDM-based wireless system is significantly degraded in the presence of nonlinear HPA. A lot of literature has been focused on reducing the peak power of the signal which in turn helps in mitigating the nonlinear distortion caused by the HPA. The PAPR reduction techniques proposed in the literature can be broadly classified into three groups: Signal distortion techniques, multiple signaling and probabilistic (MSP) techniques, and coding techniques [14–16]. Signal distortion techniques pre-distort the OFDM signal which reduces the PAPR and then pass it through the HPA. However, signal distortion technique introduces both in-band and out-of-band distortion which leads to an increase in bit error rate (BER). MSP techniques modify the OFDM signal by adding peak reduction carriers, introducing phase shifts, or changes in constellation points. These techniques reduce the PAPR at the expense of high implementation complexity and transmit power [14, 15]. Coding techniques such as linear block codes, Golay sequences, etc., also reduces the PAPR of the OFDM signal but it also reduces the bit rate of the system [15, 16]. The comparison of different PAPR reduction techniques are summarized in Table 2.1.

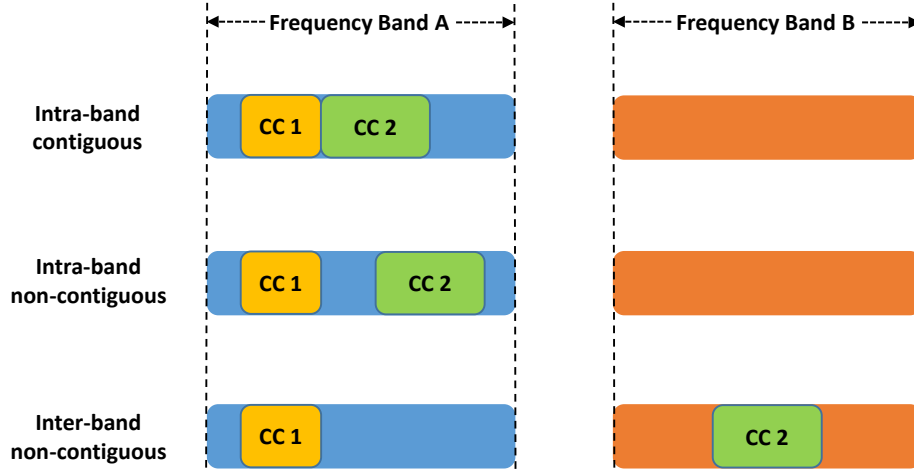


Figure 2.1: Different types of Carrier Aggregation

2.1.2 Carrier Aggregation

CA, also known as spectrum aggregation, which has been incorporated as one of the distinctive features of LTE-A standard was proposed to enhance the data rates of existing LTE systems. It allows scalable widening of effective bandwidth by combining the multiple unused component carriers (CCs) to support wideband signals [6, 7]. These CCs may be in the same or different bands and may be of different bandwidths to provide maximum flexibility in utilizing the scarce radio spectrum. There are three types of CA depending on the continuity and positioning of the CC: Intra-band contiguous (CCs are continuous and present in same frequency band), Intra-band non-contiguous (CCs are non-continuous but present in same frequency band), and Inter-band non-contiguous (CCs are present in different frequency bands) as shown in Fig. 2.1 [17]. It also provides better resource utilization, spectral efficiency and increased flexibility in terms of load sharing among carriers and frequency allocation [18–20]. In 4G LTE-A, up to five CCs each having a maximum bandwidth of 20MHz can be aggregated together to get wide bandwidth of maximum 100MHz, while in LTE-A Pro (4.5G, comprising LTE Releases 13 and 14), beyond five CCs for CA are considered and up to 32 CCs can be aggregated which is known as massive or enhanced CA (MCA or eCA) [21].

2.1.3 Multi-User MIMO

MIMO plays a vital role in enhancing network coverage, system throughput, and transmission reliability. With the advent of the advanced MIMO techniques, it has become one of the key features of LTE-A standard [22]. A MU-MIMO system, which is a subset of massive MIMO [23]), can simultaneously serve multiple users by spatially sharing the channel in contrast to

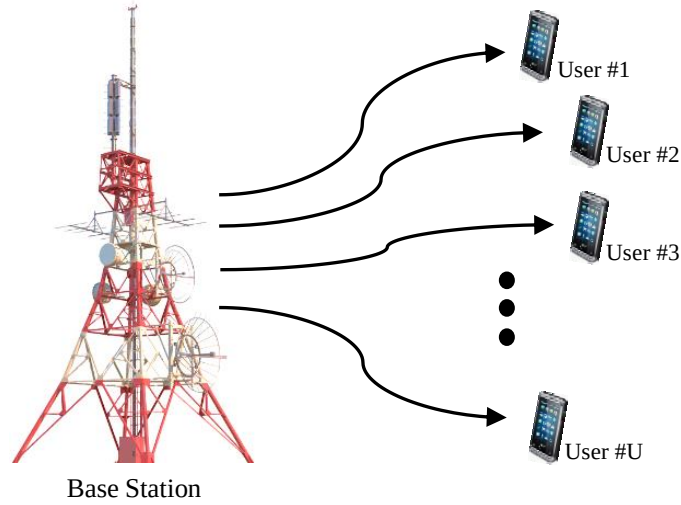


Figure 2.2: Schematic of Downlink MU-MIMO System

single-user (SU) MIMO system [24–26]. In MU-MIMO, a base station (BS) having multiple antennas utilizes transmit precoding schemes to facilitate simultaneous communication to several mobile users as shown in Fig. 2.2. Hence, it can attain high capacity and multiplexing gains by sharing the same time-frequency resource among multiple users without expanding the network bandwidth. From a practical perspective, MU-MIMO is easy to implement and cost-efficient as it requires upgrading the BS rather than numerous mobile user equipment [24, 25]. With respect to SU-MIMO, MU-MIMO is more immune to limitations like channel rank loss and degradation caused by a line-of-sight (LOS) propagation [26]. Owing to its aforementioned benefits, MU-MIMO technique has gained considerable popularity; thus, making it an indispensable contender technology for realizing fifth generation (5G) standards [27, 28]. Although MU-MIMO has several advantages over SU-MIMO, a multi-user interference¹ (MUI) severely limits the performance gains of MU-MIMO. Since BS broadcasts the data signals intended to all users simultaneously, each user receives its own desired signal as well as other users' signals which act as an interference to that user and is referred to MUI. Several linear and nonlinear precoding schemes have been proposed in the literature to suppress or eliminate MUI and to improve the MU diversity, prominent among them are channel inversion [29], block diagonalization [30], transmit preprocessing technique [31], perturbation [32], and dirty paper coding [33].

¹A multi-user interference is also known as inter-user interference. Both terms can be used interchangeably.

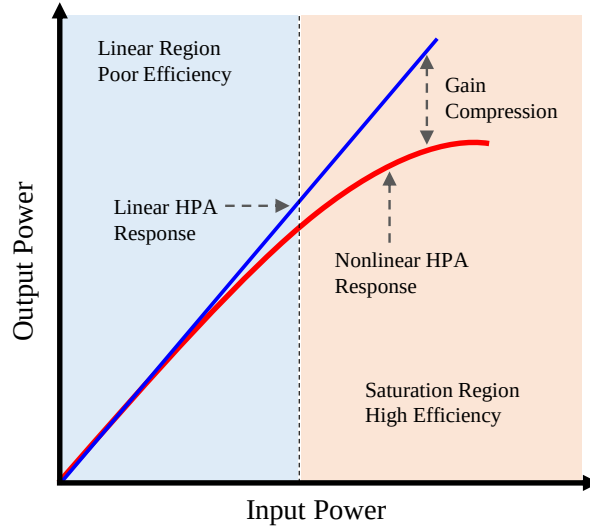


Figure 2.3: Input-output characteristics of a linear and nonlinear HPA.

2.1.4 Hardware Impairments

Wireless communication system consists of several hardware that play a vital role in the data transmission. All the hardware are made of electronic components such as resistors, inductors, capacitors, diodes, and transistors, which are made up of conductor and semiconductor materials. These components are not perfect and can behave differently from their expected nature due to several reasons such as temperature variation, aging, pressure, mismatch between the components, etc. Hardware impairments are an inevitable consequence of using non-ideal components in wireless communication systems. HPA nonlinearities, I/Q imbalance, quantization noise, jitter in digital sampling clock, oscillator phase noise, and MIMO crosstalk are examples of such impairments [34, 35]. In this dissertation, the focus is on HPA nonlinearities and oscillator phase noise which are the major hardware impairments affecting the performance of wireless communication systems.

2.1.4.1 High Power Amplifier

HPAs are one of the essential components in wireless communication systems. They provide enough output power, typically in BSs, for reliable and long distance communication. Theoretically, the HPAs are assumed to be linear, or at least linear within the dynamic range of the transmitted signal while analyzing the performance of a wireless communication system. However, various studies have shown this may not be true for multi-carrier wideband systems [10–13]. HPAs are conventionally designed to operate near saturation region to attain maximum efficiency. A typical input-output characteristics of a HPA is shown in Fig. 2.3. When HPA is

driven with multi-carrier wideband signals with high PAPR, it introduces a significant in-band distortion and out-of-band distortion which results in reduced reliability and high ACPR, respectively. These distortions degrade the quality of a transmitted signal, and as a consequence, deteriorate the overall system performance of wireless networks. Hence, it is imperative to take into account the impact of nonlinear transmitters while evaluating the performance of the wireless networks.

Behavioral modeling allows us to mathematically relate the input and output of the device under test (HPA in our case) [36, 37]. Here, the modeled device is considered as a “black-box”, i.e., the knowledge of the internal structure of the device does not require. Behavioral modeling can help to identify the nonlinear distortion effects caused by HPA which makes it a time and resource efficient process for transceiver performance evaluation. Moreover, it is also helps in the design of DPD techniques. Numerous models have been proposed for behavioral modeling of nonlinear HPAs. These models can be classified in two groups: memoryless models with static nonlinearity and memory models with dynamic distortions. Memoryless models depend only on the current input signal to the HPA and capture an amplitude-to-amplitude modulation (AM/AM) and amplitude-to-phase modulation (AM/PM) conversions of nonlinear HPA [38]. The commonly used memoryless models are Rapp model [39], Saleh model [40] and polynomial model [41].

On the other hand, memory models which depend on the current as well as the previous input signals to the HPA, characterizes the nonlinearity as well as the memory effects of HPAs [42–44]. Several behavioral models such as Wiener, Hammerstein, Wiener-Hammerstein, parallel Wiener, memory polynomial (MP), generalized memory polynomial (GMP) have been conventionally used in literature to characterize the nonlinear behavior of HPAs [9, 44, 45]. Through judicious selection of parameters, these models were able to capture the in-band and out-of-band distortions caused due to nonlinear HPA. Most commonly used among them is MP model [43] which can be expressed as

$$y[n] = \sum_{p=1}^P \sum_{m=0}^{M-1} c_{p,m} x[n-m] |x[n-m]|^{p-1}, \quad (2.1)$$

where $x[n]$ and $y[n]$ are the input and output of the nonlinear HPA, respectively, and $c_{p,m}$ are the complex nonlinearity coefficients of the HPA for p^{th} order nonlinearity of the m^{th} memory depth. P represents the total nonlinearity order and M is the memory depth. A more generalized form of MP model is proposed in [44] where direct way of adding cross terms, i.e., the product terms with different timeshifts to an MP model is introduced without complicating

the extraction procedure, known as GMP model. The GMP model can be expressed as

$$y[n] = \sum_{p=1}^P \sum_{m=0}^{M-1} \sum_{g=-G}^G c_{p,m,g} x[n-m] |x[n-m-g]|^{p-1}, \quad (2.2)$$

where $c_{p,m,g}$ are the complex nonlinearity coefficients of the HPA for p^{th} order nonlinearity of the m^{th} memory depth with g^{th} cross term. G is the number of cross terms. On substituting the $P = 1$, $M = 1$, $G = 0$ and $c_{1,0,0}=1$, the expression in (2.2) is reduced to $y[n]=x[n]$ which is a linear case. These models are generally used to evaluate the performance of various wireless communication system in the presence of nonlinear HPAs. To reduce these distortions, HPAs can be operated with high IBO; however, it compromises their power efficiency.

In this dissertation, the different behavioral models are used to represent the nonlinear characteristics of HPA in different chapters. In Chapter 3 and 4, the GMP model is used which is most accurate behavioral model to characterized the dynamic distortions. In chapter 5 and 6, the memoryless models are used to characterized the static nonlinearity. For ease of mathematical analysis, memoryless models are used in the latter chapters; however, the results of the latter chapters can be reproduced using the other HPA models.

2.1.4.2 Phase Noise

Oscillators are one of the main building blocks in the wireless communication system. They are used to up-convert/down-convert the baseband signal to/from RF signal. Unfortunately, oscillators suffer from hardware imperfections that introduce phase noise to the wireless communication system. A perfect oscillator would have localized tones at discrete frequencies, but any corrupting noise spreads these perfect tones, resulting in high power levels at neighboring frequencies. Phase noise severely limits the performance of systems that employ dense constellations due to decision errors by moving the received signals to wrong decision regions. It also destroys the orthogonality of the subcarriers in OFDM systems and causes ICI. These effects significantly degrade the performance of the wireless communication systems [46, 47].

2.1.5 Predistortion

The HPA are generally operated close to saturation region for high power efficiency. However, it introduces the nonlinear distortion which results in degradation of the system performance especially in case of high PAPR signals such as OFDM. A lot of literature has been contributed towards the mitigation of nonlinear distortion caused by HPA. Generally, literature follows two class of techniques: PAPR reduction methods and linearization methods. Various PAPR reductions techniques are already discussed in Section 2.1.1. These techniques focus on

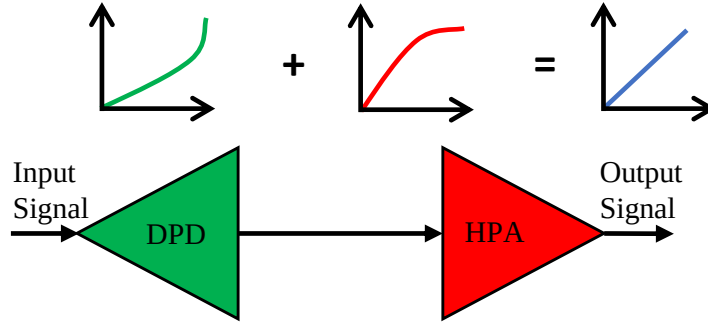


Figure 2.4: A characteristics of DPD and HPA at the transmitter.

reducing the peak power of the signal which helps in operating the HPA towards linear region. Furthermore, HPA can be forced to operate with high back-off to obtain crest reduced signals [48] which results in a reduction of the power efficiency of the HPA. Another way to deal with a low power efficiency of the HPA is envelope tracking (ET). It tunes the voltage bias inside the HPA according to the input signal which leads to the high power efficiency of the HPA [49]. However, it degrades in-band transmit signal quality and causes spectral regrowth [50].

Another way to mitigate spectral regrowth and nonlinear distortion is linearization techniques. Numerous authors have proposed many linearization techniques to increase the efficiency and reduce the distortions caused due to HPA. These techniques are restricted to using either analog methods [51–53] such as analog predistortion, feed-forward linearization, amplitude and phase feedback, envelope elimination and restoration etc., or digital methods [9, 44, 54–57] such as DPD, postdistortion etc. Among them, DPD is predominantly used due to its reconfigurable and flexible implementation and ability to improve the system performance [54]. Furthermore, DPD has also been used to mitigate other distortions such as MIMO crosstalk [58–61]. DPD distorts the transmitted signal in such a manner that the ideal cascaded DPD+HPA output will be linear (ideally) in nature as shown in Fig. 2.4. It has inverse nonlinear characteristics as compared to the HPA, and it is placed before the HPA in the transmitting chain [62]. It significantly suppresses the spectral regrowth and improves the transmission reliability.

2.2 Related Work

MIMO-OFDM is the most important feature of LTE-A standard. By utilizing the benefits of MIMO and OFDM together, the performance of wireless communication systems can be improved significantly. However, OFDM is highly sensitive to nonlinear HPAs due to its high PAPR. Nonlinear HPAs have been shown to degrade the performance of OFDM based wireless

systems considerably. Likewise, the MIMO-OFDM system is also greatly affected by HPA nonlinearity. Hence, it is important to take into account the impact of nonlinear HPA while evaluating the performance of the OFDM-based systems.

Sufficient amount of research has been devoted to studying the effects of nonlinear distortion caused by HPA when driven with a high PAPR signal. For example, authors in [63] evaluated the capacity performance of a clipped-OFDM. In [64], the analysis of HPA nonlinearity was based on the application of Bussgang's theorem with complex Gaussian input signals. Authors in [65, 66] used the statistical approach to model the nonlinear distortion caused by HPA. The above analyses were also extended for multi-antenna systems. For instance, the error performance of a single-carrier MIMO system with transmit beamforming was analyzed in the presence of nonlinear HPAs in [67]. The impact of nonlinear HPAs on MIMO-OFDM system with zero-forcing (ZF) equalization was investigated in [68, 69] and MIMO-OFDM system with single value decomposition (SVD) in [70]. These analyses had shown that the nonlinear distortions cause an irreducible error floor in high SNR regime and also degrade the system capacity. Furthermore, these analyses also showed the effect of a number of transmit antennas on the nonlinear distortion caused by HPA.

Recent studies have shown that when two or more OFDM signals are aggregated together (i.e., CA-OFDM), the resultant PAPR of the time domain signal is significantly higher than the PAPR of individual OFDM signals when there is a single HPA present in the transmitting chain [71–75]. This increment in PAPR is due to constructive adding of data across the aggregated bands. Further, when high PAPR CA-OFDM signal passes through a nonlinear HPA, the signals from one component carrier may interfere with other which introduce the cross-modulation products in addition to in-band and out-of-band modulation products [76, 77]. These terms cause more severe nonlinear distortion as compared to the non-aggregated/single-band (SB) OFDM system. The conventional models are inadequate to characterize the cross-modulation effects. As a consequence, the conventional analysis on a SB nonlinear OFDM system [63–70] may not be applicable to CA-OFDM system. Several researchers proposed new models to capture the nonlinearity of dual-band HPAs which includes two-dimensional (2D) enhanced Hammerstein (EH) [76] and 2D-MP model [77]. Authors in [78] investigated the impact of cross-modulation effects using the 2D-MP model on the DB-MIMO-OFDM system. They analytically evaluated the error performance of the DB-MIMO-OFDM system in the presence of nonlinear HPAs. However, the proposed behavioral models are restricted in a sense that they can only be used to model the nonlinear distortion of HPA when excited by DB CA-OFDM signal. Moreover, researchers give more focus towards investigating the spectral regrowth and ACPR while analyzing the DB behavioral models without giving much consideration to

the reliability of wireless communication systems. Thus, it is important to address the above mentioned problems which are not investigated thoroughly in the literature.

Incorporating OFDM with other advance wireless techniques has been shown further improvement in the performance of wireless networks. For instance, by exploiting the benefits of MU-MIMO and OFDM together, the efficiency of the wireless networks can be improved significantly. This merger of OFDM and MU-MIMO systems is commonly known as MU-MIMO-OFDM systems [79]. Several researchers have analyzed these systems and propose various techniques to improve their performance. For example, authors in [80] evaluated the performance of distributed MU-MIMO-OFDM system using greedy MU scheduler. In [81], authors analyzed the multi-cell MU-MIMO-OFDM systems using various channel estimation schemes. Furthermore, a novel frame structure and scheduling algorithm for MU-MIMO-OFDM system are proposed to evaluate the average cell throughput in [82]. Due to its superiority over conventional MIMO-OFDM, MU-MIMO-OFDM has also been incorporated in prominent wireless standards such as IEEE 802.11ac, LTE-A, and 5G new radio (NR) [83]. Furthermore, the application of CA in MU-MIMO-OFDM system increases the overall capacity of wireless networks by manifolds. In [84], authors have studied the joint implementation of MU-MIMO-OFDM and CA schemes in wireless networks. They showed that this joint implementation boosts the throughput performance of wireless networks. Despite these attributes, it also suffers from the high PAPR problem which makes it susceptible to nonlinear distortion caused due to HPA. The nonlinear distortion significantly reduces the system reliability and capacity. A few works have been devoted to reducing the PAPR of single-carrier/non-aggregated MU-MIMO-OFDM systems [85–87]. However, the end-to-end analytical performance of CA MU-MIMO-OFDM systems in the presence of HPAs is not investigated in the literature.

All the prior work was limited to evaluating the system performance in the presence of nonlinear HPAs, without much consideration on other hardware impairments such as phase noise due to non-ideal oscillators. Phase noise is caused by the fluctuation in local oscillators present at the transmitter and receiver. It significantly degrades the efficiency and performance of the system [88]. Hence, it is imperative to evaluate the tolerance of a communication system to any change in the phase noise. Several researchers have investigated the impact of phase noise on the OFDM systems [47, 88–91]. It has been shown that phase noise causes a CPE and ICI. CPE acts as a phase rotation factor which alters the phase of the complex data symbols of the OFDM subcarriers and does not vary with the OFDM symbol period. In addition, the phase noise also disrupts the orthogonality between the subcarriers which leads to ICI. If not adequately compensated, this can cause considerable degradation in the performance of the OFDM systems. The impact of phase noise has also been investigated for different MIMO systems. For instance, the performance of uplink MU-MIMO system with

maximal ratio combining (MRC) and massive MIMO with a regularized ZF precoder in the presence of phase noise has been studied in [92] and [93], respectively. These works have been extended for multi-carrier modulation schemes in [94, 95]. They have shown significant degradation in a SINR of the MU-MIMO system in the presence of phase noise, thus restricting the gains of the MU-MIMO system. Some of the recent works have also investigated the joint effects of phase noise and HPA nonlinearity on OFDM-based systems [96–99]. Specifically, they evaluated the error performance of OFDM system in the presence of both nonlinear HPA and phase noise for an additive white Gaussian noise (AWGN) channel. Authors in [100] extended the above analysis for the MIMO-OFDM systems in Rayleigh fading channel. They showed the effects of nonlinear distortion and phase noise in terms of frame error rate. However, [100] did not analytically evaluate the impact, and the results were shown through the simulations only. Furthermore, the performance of MU-MIMO-OFDM system in the presence of both nonlinear distortion and phase noise is not reported in the literature. It is also worth noting that all aforementioned theoretical analysis was restricted to analyzing the performance due to nonlinear HPA only, without much consideration on end-to-end performance in the presence of both DPD and HPA. In addition, the performance with respect to DPD has been limited to measuring the spectral regrowth and ACPR which arises due to the out-of-band emissions. Furthermore, these performances have been evaluated via computer simulations only [54, 56, 62]. However, the system reliability which generally depends on the in-band distortion has not been analyzed in the presence of the DPD for the nonlinear multi-carrier wideband communication systems.

The shortcomings and research gaps that explained above are summarized below:

1. Both CA-MIMO-OFDM and MU-MIMO-OFDM systems have PAPR problem. Consequently, both systems suffer from the nonlinear distortion in the presence of HPA.
2. The conventional analysis on a SB/non-aggregated OFDM system may not be applicable to CA-OFDM system.
3. New behavioral models have been proposed to evaluate the performance of the nonlinear DB-MIMO-OFDM system. However, this analysis is restricted to the DB-MIMO-OFDM system, and generalized expressions for multi-band CA-MIMO-OFDM system, having more than two aggregated CCs were not derived.
4. Although the performance of linear MU-MIMO-OFDM system has been studied, we are not aware of any work that investigates the impact of transmitter nonlinearity on the performance of MU-MIMO OFDM system.

5. Most of the prior work on the analytical evaluation of the joint impact of phase noise and transmitter nonlinearity has been restricted to OFDM systems.
6. MU-MIMO-OFDM systems are fundamentally different from conventional OFDM systems, thus we believe the joint impact of the above impairments will shed interesting insights into the performance of such systems.
7. The performance with respect to DPD has been limited to measuring the spectral regrowth and ACPR via computer simulations only.
8. The system reliability has not been analyzed in the presence of the DPD for the nonlinear multi-carrier wideband communication systems.

Multi-Band Carrier Aggregated Nonlinear MIMO-OFDM System

This chapter presents the impact of nonlinear HPAs on the multi-band CA-MIMO-OFDM systems. The brief introduction and contributions related to above mentioned system are given in Section 3.1 followed by Section 3.2 which presents the system model. Section 3.3 describes the theoretical approach of CA-MIMO-OFDM system. The derivation of SER and EVM are provided in Sections 3.4 and 3.5, respectively. The numerical and simulation results are presented in Section 3.6. Finally, Section 3.7 concludes the chapter.

3.1 Introduction

CA-MIMO-OFDM system comprises three prominent features of 4G LTE-A: CA yields wide bandwidth, MIMO is used to provide diversity and multiplexing gain, and OFDM mitigates the multi-path fading. However like the conventional OFDM system, CA-MIMO-OFDM system is also vulnerable to nonlinear distortion caused by HPAs. Authors in [78] investigated the performance of CA-MIMO-OFDM system in the presence of nonlinear HPA. However, this analysis was restricted to a DB CA-MIMO-OFDM system, and generalized expressions for multi-band CA-MIMO-OFDM system, having more than two aggregated CCs, were not derived. In this chapter, we analyze the impact of nonlinear HPA on multi-band CA-MIMO-OFDM system, a topic which has been overlooked so far.

The nonlinear behavior of the HPA is modeled by a multi-band (MB) GMP model. It is shown that the received symbol after down-conversion can be canonically decomposed into complex attenuation factor and additive nonlinear noise. The generalized mathematical expressions of complex attenuation factor and additive nonlinear noise for any number of

aggregated bands with any nonlinearity order and any memory depth of HPA are derived. From these expressions, the overall performance of a multi-band CA-MIMO-OFDM system in terms of SER and EVM for M-ary QAM is evaluated. The proposed work also provides valuable insights on the impact of the number of aggregated carriers on the error performance of a nonlinear CA-MIMO-OFDM system¹. It is shown that the nonlinear noise is a function of the number of aggregated carriers and increases as the number of aggregated carriers increases. Further, this nonlinear noise also impacts the diversity order of the nonlinear CA-MIMO-OFDM system. Although the performance of the nonlinear CA-MIMO-OFDM improves by increasing diversity gain in low signal-to-noise ratio (SNR) regime, it is shown that there is no substantial impact of increasing diversity on higher SNR regime.

In addition to above, generalized expression of EVM of a nonlinear CA-MIMO-OFDM system in the multi-path Rayleigh fading channel is derived. This expression is valid for any number of aggregated bands, any number of transmit/receive antennas, any nonlinearity order or memory depth. Finally, simulation results are presented to validate the efficacy of the proposed analysis by comparing it with the analytically obtained expressions. Some of the significant contributions of the proposed work are summarized as below:

- Generalized expressions of complex attenuation factor and additive nonlinear noise for any number of aggregated bands with any nonlinearity order and any memory depth of HPA are derived.
- From the derived expressions, an analytical methodology is proposed to obtain the received signal-to-distortion-plus-noise ratio (SDNR) and SER of a nonlinear CA-MIMO-OFDM system.
- The closed-form expression for EVM at both transmitter and receiver is also derived for nonlinear CA-MIMO-OFDM system.
- Impact of the number of aggregated carriers on SER and EVM of CA-MIMO-OFDM system is investigated. It is shown that the SER and EVM degrade with the number of aggregated carriers.
- Effect of diversity order on the performance of CA-MIMO-OFDM system is investigated. Although the performance of CA-MIMO-OFDM can be improved by increasing diversity gain in low SNR regime, there is no substantial impact of increasing diversity on higher SNR regime.

¹For brevity, we denote a CA-MIMO-OFDM system in the presence of nonlinear HPA as a nonlinear CA-MIMO-OFDM system.

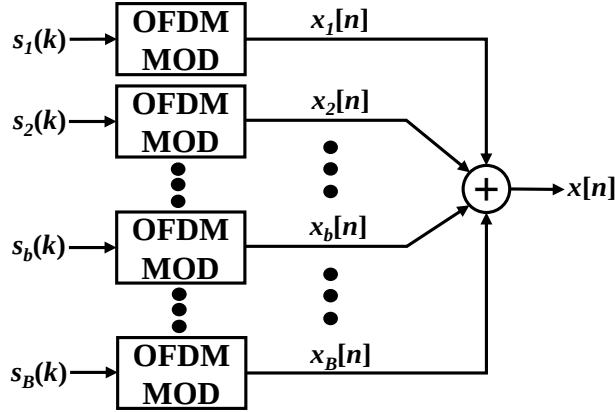


Figure 3.1: CA-OFDM signal with aggregation of B number of CCs.

3.2 System Model

This section presents the CA-OFDM system model, a behavioral model for multi-band HPAs, and multi-path Rayleigh fading MIMO channel model.

3.2.1 CA-OFDM Signal

A CA-OFDM signal consists of multiple CCs that are aggregated together to transmit wideband signals. Each CC has an OFDM modulator which modulates the incoming complex data symbols into OFDM symbols. All CCs are aggregated together using power combiner as shown in Fig. 3.1. Each OFDM modulator consists of serial-to-parallel converter, inverse discrete Fourier transform (IDFT), cyclic prefix adder and parallel-to-serial converter blocks. The baseband equivalent expression of the OFDM symbol with N subcarriers, appended with the cyclic prefix, N_g , and the discrete time index n can be represented as [41]:

$$x_b^t[n] = \frac{1}{N} \sum_{k=0}^{N-1} s_b^t(k) e^{j2\pi kn/N}, \quad -N_g \leq n \leq N-1 \quad (3.1)$$

where $x_b^t[n]$ is the OFDM symbol input to the HPA of t^{th} transmitting antenna at b^{th} CC, $s_b^t(k)$ is the complex data symbol on k^{th} subcarrier of the OFDM symbol at b^{th} CC. Without loss of generality, it is presumed that complex data symbols $s_b^t(k)$ are independent and identically distributed (i.i.d.) with zero mean and equal variance of P_s/B , where P_s is the total power in k^{th} subcarrier across all CC and B is total number of aggregated CCs. By the Central Limit Theorem (CLT) [101], when the number of subcarriers N is large, $x_b^t[n]$ can be assumed to be a complex Gaussian process with zero mean and variance $P_{in} = P_s/NB$ [41].

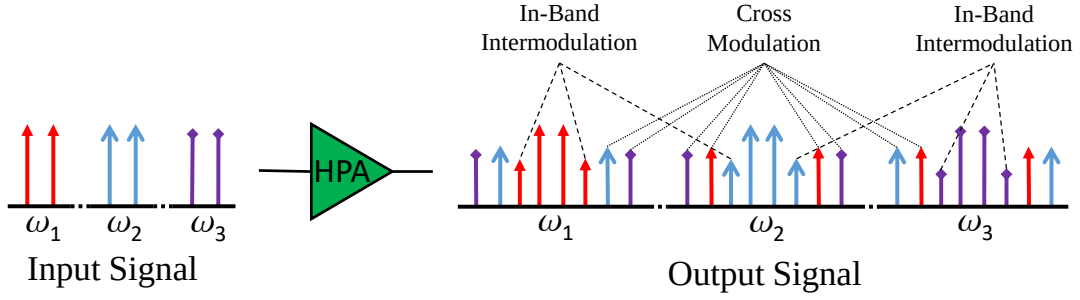


Figure 3.2: Distortion due to MB nonlinear transmitter.

3.2.2 Behavioral Model for HPA

2D-EH and 2D-MP models have been used to characterize the nonlinear distortion caused by HPAs on aggregation of dual CCs in [76] and [77], respectively. However, both models are restricted in a sense as it can be only used to model the nonlinear distortion of HPA when excited by DB CA-OFDM signal. 2D-MP can be extended for B number of bands by including more cross modulation products and on including the leading and lagging terms, more generalized model can be obtained. Thus, the resultant model is termed as MB-GMP and can be expressed as:

$$\begin{aligned}
 y_b^t[n] = & \sum_{p_1=1}^P \sum_{p_2=1}^{p_1} \cdots \sum_{p_B=1}^{p_{B-1}} \sum_{m=0}^{M-1} \sum_{g=-G}^G c_{p_1, p_2, \dots, p_B, m, g} x_b^t[n-m] |x_1^t[n-m-g]|^{p_1-p_2} \\
 & \times |x_2^t[n-m-g]|^{p_2-p_3} |x_3^t[n-m-g]|^{p_3-p_4} \cdots |x_b^t[n-m-g]|^{p_b-p_{b-1}} \\
 & \times |x_{b+1}^t[n-m-g]|^{p_{b+1}-p_b} \cdots |x_B^t[n-m-g]|^{p_B-1}, \quad (3.2)
 \end{aligned}$$

where $y_b^t[n]$ is the output of the nonlinear HPA of the t^{th} transmitting antenna corresponding to the input signal $x_b^t[n]$ in b^{th} frequency band. P denotes the nonlinearity order, M is the memory depth and G is the number of cross terms. $c_{p_1, p_2, \dots, p_B, m, g}$ are the complex nonlinearity coefficients of HPA for p_B^{th} order nonlinearity and m^{th} memory depth with g^{th} cross term [44]. It can be noticed from (3.2) that term $|x_b^t[n-m-g]|^{p_b-p_{b-1}}$ acts as an in-band intermodulation product and remaining terms such as $|x_2^t[n-m-g]|^{p_2-p_3}$, $|x_3^t[n-m-g]|^{p_3-p_4}$ and so on are the cross modulation terms. The impact of these terms on the signal at desired frequency bands in frequency domain is shown in Fig. 3.2. For ease of exposition, HPAs in all transmit antennas are assumed to have same behavioral characteristics [70]. The frequency separations between the aggregated bands are assumed to be large so that the out-of-band distortion lies far from operating frequencies and can easily be filtered out [77]. Hence, the current analysis is restricted to in-band distortions.

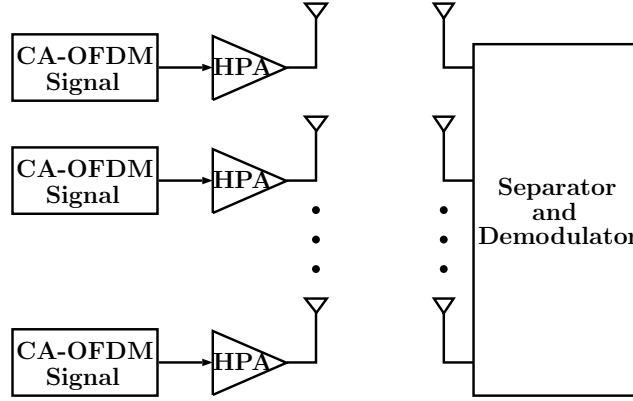


Figure 3.3: Schematic of CA-MIMO-OFDM system.

3.2.3 MIMO Channel

The MIMO system consists of N_T transmit antennas and N_R receive antennas. The channel is assumed to be a wide sense stationary uncorrelated scattering (WSSUS) Rayleigh fading channel [66]. The equivalent channel model can be represented as:

$$\bar{\mathbf{H}}[n] = \sum_{l=0}^{L-1} \bar{\mathbf{H}}^l \delta[n-l], \quad (3.3)$$

where $\bar{\mathbf{H}}[n]$ is the MIMO channel response, L is total number of delay taps, and $\bar{\mathbf{H}}^l$ contains the MIMO channel coefficients for the l^{th} tap which can be written as:

$$\bar{\mathbf{H}}^l = \begin{bmatrix} h_{1,1,l} & h_{1,2,l} & \cdot & \cdot & h_{1,N_T,l} \\ h_{2,1,l} & h_{2,2,l} & \cdot & \cdot & h_{2,N_T,l} \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ h_{N_R,1,l} & h_{N_R,2,l} & \cdot & \cdot & h_{N_R,N_T,l} \end{bmatrix}_{N_R \times N_T} \quad (3.4)$$

where $h_{i,t,l} \sim \mathcal{CN}(0,1)$ is the channel impulse response between t^{th} transmitting antenna and i^{th} receive antenna for l^{th} delay tap. We assume that the channel is perfectly known at the receiver to decode the transmit signals.

3.3 CA-MIMO-OFDM System

The schematic of CA-MIMO-OFDM system is shown in Fig. 3.3. Each CA-OFDM signal is passed through a nonlinear HPA at each transmitter. At receiver, multiple antennas receive

signals from each Tx antenna. The baseband representation of a received signal vector for b^{th} CC can be expressed as²

$$\bar{\mathbf{r}}_b[n] = \sum_{l=0}^{L-1} \bar{\mathbf{H}}^l \bar{\mathbf{y}}_b[n-l] + \bar{\mathbf{w}}[n], \quad (3.5)$$

where $\bar{\mathbf{r}}_b[n]$ is a $N_R \times 1$ vector of received signals that are present in b^{th} CC, $\bar{\mathbf{y}}_b[n]$ is a $N_T \times 1$ transmit signal vector at the output of HPA for b^{th} CC, $\bar{\mathbf{w}}[n]$ is a $N_R \times 1$ AWGN vector whose components are i.i.d. $\mathcal{CN}(0, N_o)$. The signal in i^{th} receive antenna is given as

$$r_b^i[n] = \sum_{t=0}^{N_T-1} \sum_{l=0}^{L-1} h_{i,t,l} y_b^t[n-l] + w^i[n]. \quad (3.6)$$

On substituting (3.2) into (3.6), we get

$$\begin{aligned} r_b^i[n] &= \sum_{t=0}^{N_T-1} \sum_{l=0}^{L-1} h_{i,t,l} \sum_{p_1=1}^P \sum_{p_2=1}^{p_1} \cdots \sum_{p_{B-1}=1}^{p_{B-2}} \sum_{m=0}^{M-1} \sum_{g=-G}^G c_{p_1, p_2, \dots, p_B, m, g} x_b^t[n-m-l] \\ &\quad \times |x_1^t[n-m-g-l]|^{p_1-p_2} \cdots |x_B^t[n-m-g-l]|^{p_{B-1}-1} + w^i[n]. \end{aligned} \quad (3.7)$$

After removal of cyclic prefix and applying discrete Fourier transform (DFT), the received complex data symbols at k^{th} subcarrier can be expressed as

$$\begin{aligned} r_b^i(k) &= \sum_{t=0}^{N_T-1} \sum_{l=0}^{L-1} h_{i,t,l} \sum_{n=0}^{N-1} \sum_{p_1=1}^P \sum_{p_2=1}^{p_1} \cdots \sum_{p_{B-1}=1}^{p_{B-2}} \sum_{m=0}^{M-1} \sum_{g=-G}^G c_{p_1, p_2, \dots, p_B, m, g} x_b^t[n-m-l] \\ &\quad \times |x_1^t[n-m-g-l]|^{p_1-p_2} \cdots |x_B^t[n-m-g-l]|^{p_{B-1}-1} e^{-j2\pi kn/N} + w^i(k), \end{aligned} \quad (3.8)$$

where $w^i(k)$ is the noise term at k^{th} subcarrier in i^{th} receive branch. Using circular time shift property of DFT³, (3.8) can be rewritten as

$$\begin{aligned} r_b^i(k) &= \sum_{t=0}^{N_T-1} \sum_{l=0}^{L-1} h_{i,t,l} \sum_{n=0}^{N-1} \sum_{p_1=1}^P \sum_{p_2=1}^{p_1} \cdots \sum_{p_{B-1}=1}^{p_{B-2}} \sum_{m=0}^{M-1} \sum_{g=-G}^G c_{p_1, p_2, \dots, p_B, m, g} x_b^t[n+g] \\ &\quad \times |x_1^t[n]|^{p_1-p_2} \cdots |x_B^t[n]|^{p_{B-1}-1} e^{-j2\pi k(n+m+g+l)/N} + w^i(k). \end{aligned} \quad (3.9)$$

²Although the analytical results have been obtained for b^{th} CC, however the aforementioned analysis is applicable to any CC.

³To ensure the application of circular time shift property of DFT in OFDM, the length of cyclic prefix should be greater than the total number of delay taps of channel, i.e., $N_g > L$

After rearranging (3.9), we obtain

$$r_b^i(k) = \sum_{t=0}^{N_T-1} \tilde{h}_{i,t,k} \tilde{s}_b^t(k) + w^i(k), \quad (3.10)$$

where $\tilde{h}_{i,t,k}$ is the effective channel coefficient from t^{th} transmit antenna to i^{th} receive antenna of k^{th} subcarrier and can be written as

$$\tilde{h}_{i,t,k} = \sum_{l=0}^{L-1} h_{i,t,l} e^{-j2\pi kl/N}, \quad (3.11)$$

and $\tilde{s}_b^t(k)$ is the complex data symbol affected by nonlinear distortion only, i.e.,

$$\begin{aligned} \tilde{s}_b^t(k) = & \sum_{n=0}^{N-1} \sum_{p_1=1}^P \sum_{p_2=1}^{p_1} \cdots \sum_{p_{B-1}=1}^{p_{B-2}} \sum_{g=-G}^G \alpha_{p_1,p_2,\dots,p_B,k,g} x_b^t[n+g] |x_1^t[n]|^{p_1-p_2} \cdots |x_B^t[n]|^{p_{B-1}-p_B} \\ & \times e^{-j2\pi k(n+g)/N}, \end{aligned} \quad (3.12)$$

where

$$\alpha_{p_1,p_2,\dots,p_B,k,g} = \sum_{m=0}^{M-1} c_{p_1,p_2,\dots,p_B,m,g} e^{-j2\pi km/N}. \quad (3.13)$$

Proposition 1 *The received complex data symbol $\tilde{s}_b^t(k)$ in (3.12) can be canonically decomposed as complex attenuation factor $\mu_b(k)$ multiplied to the desired data symbol $s_b^t(k)$ and additive nonlinear noise $\phi_b^t(k)$, and is given as*

$$\tilde{s}_b^t(k) = \mu_b(k) s_b^t(k) + \phi_b^t(k). \quad (3.14)$$

Proof: The nonlinear amplification of a zero-mean Gaussian signal $x[n]$ can be represented as⁴ [102]:

$$y[n] = \mu[n] \star x[n] + \phi[n], \quad (3.15)$$

where $\mu[n]$ is the complex attenuation factor, $\phi[n]$ is the nonlinear noise which is uncorrelated to the input signal $x[n]$ and $\star$ denotes convolution. On taking the DFT of (3.15), the received complex data symbol $\tilde{s}(k)$ can be expressed as:

$$\tilde{s}(k) = \mu(k) s(k) + \phi(k). \quad (3.16)$$

⁴For ease of representation, the superscript t and subscript b are dropped here.

The generalized expressions of complex attenuation factor $\mu_b(k)$ and nonlinear noise $\phi_b^t(k)$ (refer Appendix A) can be expressed as:

$$\begin{aligned} \mu_b(k) = & \sum_{p_1=1}^P \sum_{p_2=1}^{p_1} \cdots \sum_{p_B=1}^{p_{B-1}} (P_{in})^{\frac{p_1-1}{2}} \left[\alpha_{p_1, p_2, \dots, p_B, k, 0} \Gamma\left(1 + \frac{p_1 - p_2}{2}\right) \cdots \right. \\ & \times \Gamma\left(1 + \frac{p_b - p_{b+1} + 2}{2}\right) \cdots \Gamma\left(1 + \frac{p_B - 1}{2}\right) + \sum_{\substack{g=-G \\ g \neq 0}}^G \alpha_{p_1, p_2, \dots, p_B, k, g} \Gamma(2) \\ & \left. \times \Gamma\left(1 + \frac{p_1 - p_2}{2}\right) \cdots \Gamma\left(1 + \frac{p_B - 1}{2}\right) \right] \end{aligned} \quad (3.17)$$

and

$$\begin{aligned} \phi_b^t(k) = & \sum_{n=0}^{N-1} \sum_{p_1=1}^P \sum_{p_2=1}^{p_1} \cdots \sum_{p_B=1}^{p_{B-1}} \sum_{g=-G}^G \alpha_{p_1, p_2, \dots, p_B, k, g} x_b^t[n+g] |x_1^t[n]|^{p_1-p_2} \cdots |x_B^t[n]|^{p_B-1} \\ & \times e^{-j2\pi k(n+g)/N} - \mu_b(k) s_b^t(k), \end{aligned} \quad (3.18)$$

where $\Gamma(a)$ is the gamma function. It is noticeable that the value of $\mu_b(k)$ depends on both linear and nonlinear coefficients and it acts as gain of HPA, while $\phi_b^t(k)$ strictly depends on the nonlinear coefficients and acts as interference to an information signal. As mentioned earlier that the number of nonlinear coefficients (cross-modulation terms) increases with the number of aggregated bands. As a consequence, the value of $\phi_b^t(k)$ increases with the number of aggregated bands which affects the performance of CA-MIMO-OFDM system.

Proposition 2 *The nonlinear noise term $\phi_b^t(k)$ can be approximated as a complex Gaussian process $\mathcal{CN}(0, \sigma_{\phi_b^t(k)}^2)$ for a sufficiently large value of N .*

Proof: From (3.18), it can be observed that $\phi_b^t(k)$ is the sum of N identically distributed random variables at each k subcarrier, thus it is reasonable to model the nonlinear noise as a complex Gaussian random process for large value of N by CLT. The mean of the nonlinear noise is zero (refer Appendix B) and its variance is derived in Appendix C.

Using (3.10) and (3.14), the received complex data symbol at k^{th} subcarrier of i^{th} receive antenna can be written as:

$$r_b^i(k) = \sum_{t=0}^{N_T-1} \tilde{h}_{i,t,k} (\mu_b(k) s_b^t(k) + \phi_b^t(k)) + w^i(k). \quad (3.17)$$

From (3.17), it is quite obvious that the equivalent system of the k^{th} subcarrier in b^{th} frequency band is a MIMO system which can be expressed as

$$\mathbf{r}_b(k) = \mathbf{H}(k) (\mu_b(k)\mathbf{s}_b(k) + \Phi_b(k)) + \mathbf{w}(k), \quad (3.18)$$

where $\mathbf{r}_b(k)$ is a $N_R \times 1$ received symbol vector, $\mathbf{s}_b(k)$ is a $N_T \times 1$ transmit symbol vector and $\Phi_b(k)$ is a $N_T \times 1$ nonlinear noise vector at k^{th} subcarrier in b^{th} CC, $\mathbf{w}(k)$ is $N_R \times 1$ AWGN vector and $\mathbf{H}(k)$ is $N_R \times N_T$ effective channel whose components are $\tilde{h}_{i,t,k}$. We assume that the transmit vector of complex data symbols can be estimated at the receiver using ZF equalization [103] through

$$\hat{\mathbf{s}}_b(k) = \mu_b(k)\mathbf{s}_b(k) + \Phi_b(k) + \mathbf{H}(k)^\dagger \mathbf{w}(k), \quad (3.19)$$

where $\hat{\mathbf{s}}_b(k)$ is a $N_T \times 1$ vector of estimated transmitted data symbols and $\mathbf{H}(k)^\dagger$ is a pseudo inverse of the channel matrix $\mathbf{H}(k)$ at the k^{th} subcarrier.

3.4 SER Analysis

In this section, we measure the performance of CA-MIMO-OFDM system in terms of symbol error rate (SER) in the presence of multi-path Rayleigh fading channel. Using (3.14) and (3.19), the t^{th} estimated data symbol at the k^{th} subcarrier in b^{th} frequency band can be written as:

$$\hat{s}_b^t(k) = \mu_b(k)s_b^t(k) + \phi_b^t(k) + \left[\mathbf{H}(k)^\dagger \mathbf{w}(k) \right]^t, \quad (3.20)$$

where $\left[\mathbf{H}(k)^\dagger \mathbf{w}(k) \right]^t$ is t^{th} element of vector $\mathbf{H}(k)^\dagger \mathbf{w}(k)$. The SDNR for each subcarrier can be expressed as:

$$SDNR_b^t(k) = \frac{|\mu_b(k)|^2 P_s / B}{\sigma_{\phi_b^t(k)}^2 + \left[\left((\mathbf{H}(k))^H \mathbf{H}(k) \right)^{-1} \right]^t N_o} = \frac{\psi(k) |\mu_b(k)|^2 P_s / B}{\psi(k) \sigma_{\phi_b^t(k)}^2 + N_o}, \quad (3.21)$$

where $\psi(k) = \frac{1}{\left[\left((\mathbf{H}(k))^H \mathbf{H}(k) \right)^{-1} \right]^t}$ has a Chi-squared distribution with $2(N_R - N_T + 1)$ degrees of freedom and can be expressed as [103]

$$f_{\psi(k)}(\psi(k)) = \frac{\psi(k)^{(N_R - N_T)}}{\psi(k)^{(N_R - N_T + 1)} (N_R - N_T)!} e^{-\frac{\psi(k)}{\bar{\psi}(k)}}, \quad (3.22)$$

where $\bar{\psi}(k) = \mathbb{E}_{\psi(k)} [\psi(k)]$. Using (3.21), SER of the t^{th} received symbol for M-QAM at k^{th} subcarrier in b^{th} CC can be expressed as [104]

$$SER_{b,k}^t(\psi(k)) = 1 - \left(1 - \left(1 - \frac{1}{\sqrt{M}} \right) \text{erfc} \left(\sqrt{\frac{3SDNR_b^t(k)}{2(M-1)}} \right) \right)^2, \quad (3.23)$$

where $\text{erfc}(\cdot)$ is an error function [104]. Therefore, the SER averaged across the fading channel can be expressed as:

$$SER_{b,k}^t = \mathbb{E} [SER_{b,k}^t(\psi(k))] = \int_0^\infty SER_{b,k}^t(\psi(k)) f_{\psi(k)}(\psi(k)) d\psi(k). \quad (3.24)$$

The average SER across all the subcarriers and all transmitting branches in b^{th} CC thus can be calculated as:

$$SER_b = \frac{1}{N_T N} \sum_{t=0}^{N_T-1} \sum_{k=0}^{N-1} SER_{b,k}^t. \quad (3.25)$$

The analytical expression (3.24) is intractable and has to be evaluated using numerical methods⁵. However, for large M and moderate to high SNR, (3.24) can be bounded as [104]:

$$SER_{b,k}^t(\psi(k)) \approx 2 \left(1 - \frac{1}{\sqrt{M}} \right) \text{erfc} \left(\sqrt{\frac{3SDNR_b^t(k)}{2(M-1)}} \right) \quad (3.26)$$

Using (3.21), (3.22) and (3.26), the SER averaged across the fading channel in (3.24) can be rewritten as:

$$SER_{b,k}^t \approx A \int_0^\infty \text{erfc} \left(\sqrt{\frac{C\psi_k}{D\psi_k + 1}} \right) \psi_k^{(N_R - N_T)} e^{-\frac{\psi_k}{\bar{\Psi}_k}} d\psi_k, \quad (3.27)$$

where $A = \frac{2 \left(1 - \frac{1}{\sqrt{M}} \right)}{\Psi_k^{(N_R - N_T + 1)} (N_R - N_T)!}$, $C = \frac{3|\mu_b(k)|^2 P_s/B}{2(M-1)N_o}$ and $D = \frac{\sigma_{\phi_b^t(k)}^2}{N_o}$. Using the series expansion of the complimentary error function [105, 3.321.1], i.e., $\text{erfc}(x) = 1 - \frac{2}{\sqrt{\pi}} \sum_{f=0}^\infty \frac{(-1)^f x^{2f+1}}{f!(2f+1)}$, (3.27) can be further expressed as:

$$SER_{b,k}^t = A \left[1 - \frac{2}{\sqrt{\pi}} \sum_{f=0}^\infty \frac{(-1)^f C^{f+1/2}}{f!(2f+1)} \int_0^\infty \frac{\psi_k^{(N_R - N_T + f + 1/2)} e^{-\frac{\psi_k}{\bar{\Psi}_k}}}{(D\psi_k + 1)^{f+1/2}} d\psi_k \right] \quad (3.28)$$

⁵Several software are available to compute the expression given in (3.24) such as MATLAB, Mathematica and MAPLE. Throughout this dissertation, the ‘integral’ function of MATLAB is used to evaluate this kind of expression.

On substituting $D\psi_k = \theta_k$ in (3.28), we obtain

$$\begin{aligned} SER_{b,k}^t &= A \left[1 - \frac{2}{\sqrt{\pi}} \sum_{f=0}^{\infty} \frac{(-1)^f C^{f+1/2}}{f!(2f+1) D^{N_R-N_T+f+3/2}} \int_0^{\infty} \frac{\theta_k^{(N_R-N_T+f+1/2)} e^{-\frac{\theta_k}{D\Psi_k}}}{(\theta_k+1)^{f+1/2}} d\theta_k \right] \\ &= A \left[1 - \frac{2}{\sqrt{\pi}} \sum_{f=0}^{\infty} \frac{(-1)^f C^{f+1/2}}{f!(2f+1)} \frac{\Gamma(N_R-N_T+f+3/2)}{D^{N_R-N_T+f+3/2}} \right. \\ &\quad \left. \times U\left(N_R-N_T+f+3/2; N_R-N_T+2; \frac{1}{D\Psi_k}\right) \right], \end{aligned} \quad (3.29)$$

where $U(\cdot, \cdot, \cdot)$ is the confluent hypergeometric function [105, 9.211.4] which is given as:

$$U(a, b, c) = \frac{1}{\Gamma(a)} \int_0^{\infty} e^{-ct} t^{a-1} (1+t)^{b-a-1} dt. \quad (3.30)$$

As a special case of linear MIMO-OFDM system, it is assumed that a linear HPA is present at each transmit antenna. Due to linear HPAs, the complex attenuation factor becomes unity, i.e., $\mu_b(k) = 1$ and variance of nonlinear noise becomes zero, i.e., $\sigma_{\phi_b^t(k)}^2 = 0$. Using these values, (3.27) can be evaluated as:

$$SER_{b,k}^t \approx A \int_0^{\infty} \text{erfc}\left(\sqrt{\hat{B}}\psi_k\right) \psi_k^{(N_R-N_T)} e^{-\frac{\psi_k}{\Psi_k}} d\psi_k, \quad (3.31)$$

where $\hat{B} = \frac{3P_s/B}{2(M-1)N_o}$. On substituting $\psi_k = \theta_k^2$ in (3.31) and using [106, 4.3.8], we obtain

$$\begin{aligned} SER_{b,k}^t &= A \left[1 - 2 \int_0^{\infty} \text{erf}\left(\sqrt{\hat{B}}\theta_k\right) \theta_k^{2(N_R-N_T)+1} e^{-\frac{\theta_k^2}{\Psi_k^2}} d\theta_k \right] \\ &= A \left[1 - 2 \sqrt{\frac{\hat{B}}{\pi}} \Psi_k^{2(N_R-N_T)+3} \Gamma(N_R-N_T+3/2) {}_2F_1\left(1/2, N_R-N_T+3/2; 3/2; -\hat{B}\Psi_k^2\right) \right], \end{aligned} \quad (3.32)$$

where ${}_2F_1(\cdot)$ is hypergeometric function [105, 9.14.1].

It is a well-known fact that SER improves with an increase in AWGN SNR and diversity gain for the case of linear HPA. This fact can also be observed from (3.32). When the diversity gain, i.e., $N_R - N_T + 1$ increases, $\Gamma(\cdot)$ and ${}_2F_1(\cdot)$ terms in (3.32) also increase which results in improvement of SER. However, due to the impact of nonlinear HPA, it can be observed from (3.21) that SDNR is affected by two type of noises: 1) AWGN and 2) Nonlinear noise. In low AWGN SNR regime, AWGN dominates the nonlinear noise ($\sigma_{\phi}^2 \ll N_o$) and the system

performance improves with an increase in SNR and diversity gain. In high AWGN SNR regime, nonlinear noise dominates the AWGN ($\sigma_\phi^2 \gg N_o$) which leads to the irreducible error floor. This section provides the complete analysis of CA-MIMO-OFDM system in the presence of nonlinear HPA in terms of SER over the multi-path Rayleigh fading channel⁶.

3.5 EVM Analysis

In this section, the performance of CA-MIMO-OFDM system has been analyzed in terms of EVM. Since, the nonlinearity causes rotation and compression/expansion of the signal constellation, so EVM can be used as an alternative performance metric as it has an information about the signals phase and amplitude errors [107–110]. EVM is basically a measure of errors between the received symbols and expected symbols. It quantifies the error in the departure of signals constellation from its ideal reference because of nonlinearity. Along with SER, it is an important performance metric of wireless communication system. Further, EVM requires less complex processing than SER because to calculate SER, the received signal has to follow the whole receiver process but in case of EVM, it can be calculated before demodulation. EVM can be defined in terms of SDNR as [109]

$$EVM = \frac{1}{\sqrt{SDNR_b^t(k)}}. \quad (3.33)$$

Using (3.21) and (3.33), EVM for each subcarrier in b^{th} CC of CA-MIMO-OFDM system with nonlinear HPA in the presence of multi-path Rayleigh fading channel can be written as:

$$EVM_{b,k}^t(\psi(k)) = \sqrt{\frac{\psi(k)\sigma_{\phi_b^t(k)}^2 + N_o}{\psi(k)|\mu_b(k)|^2 P_s/B}}. \quad (3.34)$$

For each subcarrier, EVM is obtained by averaging across Rayleigh fading channel and can be expressed as

$$EVM_{b,k}^t = \int_0^\infty EVM_{b,k}^t(\psi(k)) f_{\psi(k)}(\psi(k)) d\psi(k). \quad (3.35)$$

Using (3.34) and (3.35), we have

$$EVM_{b,k}^t = \sqrt{\frac{\sigma_{\phi_b^t(k)}^2}{|\mu_b(k)|^2 P_s/B}} I_b^t(k), \quad (3.36)$$

⁶Since the closed-form expression for SER is derived for high SNR values and not applicable for low SNR values, the SER is calculated numerically using (3.24) for low SNR values.

where

$$I_b^t(k) = \int_0^\infty \sqrt{\frac{1 + \psi(k)\sigma_{\phi_b^t(k)}^2/N_o}{\psi(k)\sigma_{\phi_b^t(k)}^2/N_o}} p_{\psi(k)}(\psi(k)) d\psi(k). \quad (3.37)$$

On substituting $\psi(k)\sigma_{\phi_b^t(k)}^2/N_o = \beta$, (3.37) can be rewritten as:

$$\begin{aligned} I_b^t(k) &= \frac{1}{(N_R - N_T)! \left(\bar{\psi}(k)\sigma_{\phi_b^t(k)}^2/N_o \right)^{N_R - N_T + 1}} \times \int_0^\infty (\beta + 1)^{\frac{1}{2}} \beta^{N_R - N_T - \frac{1}{2}} e^{-\frac{\beta N_o}{\bar{\psi}(k)\sigma_{\phi_b^t(k)}^2}} d\beta \\ &= \frac{1}{\Gamma(N_R - N_T + \frac{1}{2}) (N_R - N_T)! \left(\bar{\psi}(k)\sigma_{\phi_b^t(k)}^2/N_o \right)^{N_R - N_T + 1}} \\ &\quad \times U\left(N_R - N_T + \frac{1}{2}, N_R - N_T + 2, \frac{N_o}{\bar{\psi}(k)\sigma_{\phi_b^t(k)}^2}\right). \end{aligned} \quad (3.38)$$

Using (3.36) and (3.38), the closed form expression of average EVM in b^{th} CC across all the subcarriers and all transmitting branch can be given as:

$$\begin{aligned} EVM_b &= \frac{(N_o)^{N_R - N_T + 1}}{\Gamma(N_R - N_T + \frac{1}{2}) (N_R - N_T)! N_T N} \\ &\quad \times \sum_{t=0}^{N_T - 1} \sum_{k=0}^{N - 1} \frac{U\left(N_R - N_T + \frac{1}{2}, N_R - N_T + 2, \frac{N_o}{\bar{\psi}(k)\sigma_{\phi_b^t(k)}^2}\right)}{\sqrt{\bar{\psi}(k)P_s/B} |\mu_b(k)| \left(\bar{\psi}(k)\sigma_{\phi_b^t(k)}^2 \right)^{N_R - N_T + \frac{1}{2}}}. \end{aligned} \quad (3.39)$$

3.6 Simulation Results

In this section, simulation results are presented to evaluate the performance of CA-MIMO-OFDM system impaired by nonlinear HPA with memory. As discussed before, the propagation channel is modeled as a multi-path Rayleigh fading channel. The simulations are also compared with theoretical results to validate the analytically obtained expressions. Specifically, the performance is evaluated by calculating the signal-to-distortion ratio (SDR), SER and EVM of the received CA-OFDM signal. Further, the impact of the number of aggregated bands on the diversity order on CA-MIMO-OFDM system is also investigated.

The nonlinear HPA is modeled by MB-GMP model with 3^{rd} order nonlinearity ($P = 3$), $M = 4$ and $G = 2$. For ease of simulations, it is assumed that HPAs in all transmitters have same behavioral characteristics [70]. The performance of a communication system in the presence

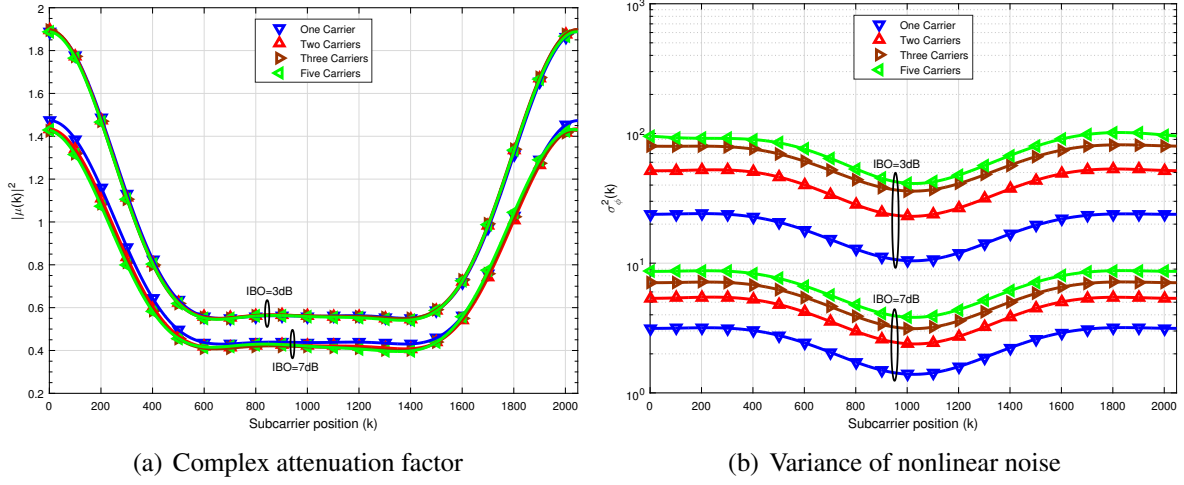


Figure 3.4: Analytical (solid lines) and simulation (marker points) results of the complex attenuation factor and variance of nonlinear noise for each subcarrier of different CA-OFDM signals for 16 QAM.

of nonlinear HPA is usually quantified by IBO, i.e., $IBO = \frac{A^2}{P_{in}}$ where A is the input saturation voltage of HPA [41]. Thus, higher the IBO, less is the impact of nonlinear distortions as HPA tends to operate near the linear region. In the following, the system performance has been shown for different values of IBO. To verify the analytical results, different sets of coefficients for different IBO are used to characterize the nonlinear HPA with memory⁷.

3.6.1 SDR Simulation

In this section, we analyze the impact of nonlinear HPA on CA-OFDM signals in the absence of channel fading and AWGN effects, i.e., $N_o = 0$. This leads to $SDR_{b,k} = \frac{|\mu_b(k)|^2 P_s / B}{\sigma_{\phi_b^t(k)}^2}$.

Fig. 3.4 shows the impact of $\mu_b(k)$ and $\sigma_{\phi_b^t(k)}^2$ on different subcarriers of a CA-OFDM system. The theoretical and simulation results coincide at each subcarrier, which validates the derived mathematical expressions. It can also be observed that both $\mu_b(k)$ and $\sigma_{\phi_b^t(k)}^2$ are a function of number of aggregated bands. As the number of aggregated bands increases in CA-OFDM signal, $\mu_b(k)$ decreases whereas $\sigma_{\phi_b^t(k)}^2$ increases which deteriorate the received SNR. This fact supports the discussion in Section 3.3.

From Fig. 3.4(a), it can also be observed that with an increase in IBO, the value of $\mu_b(k)$ increases. This is due to the fact that the increase in IBO forces HPA to operate in a linear region which increases the value of linear coefficients and reduces the value of nonlinear

⁷The number of coefficients for each IBO is between 40-120 depending upon the number of aggregated carriers and HPA parameters. Interested readers can refer https://www.iiitd.edu.in/~wirocomm/resources/coefficients_MB_GMP.xlsx

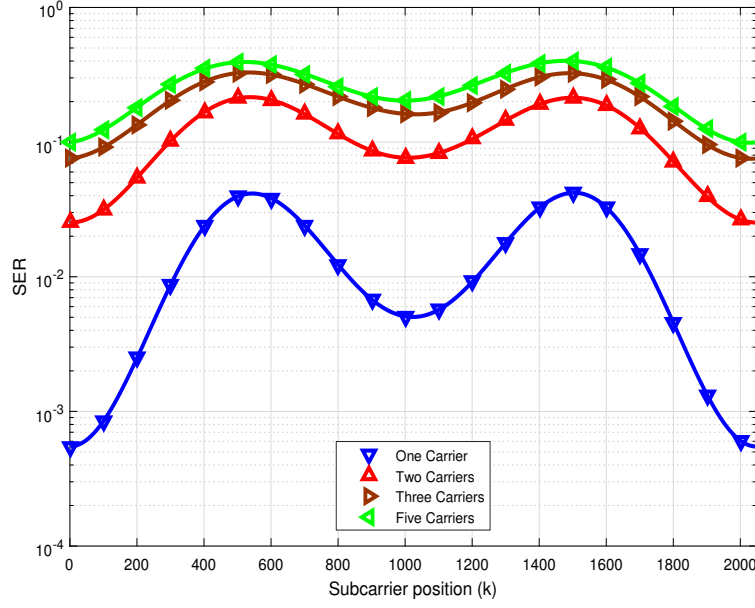


Figure 3.5: Analytical (solid lines) and simulation (marker points) results of average SER for each subcarrier of different CA-OFDM signals without AWGN for 16 QAM and IBO=3dB.

coefficients. As a result, the impact of in-band and cross-modulation terms reduces which improves the system performance. Moreover, at high IBO (say 7dB), the value of $\mu_b(k)$ for a different number of aggregated bands are approximately same because of the low impact of nonlinear coefficients.

It is also noticeable from Fig. 3.4 that the complex attenuation and variance of nonlinear noise affects each subcarrier differently. The subcarriers located towards the end of the spectrum have considerable attenuation than other subcarriers while the variance of nonlinear noise is less in middle subcarriers than others, due to which middle and edge subcarriers are less vulnerable to nonlinear distortion. Fig. 3.5 shows the impact of nonlinear distortion on the SER of each subcarrier of CA-OFDM signals when $N_o = 0$. From Fig. 3.5, it can be observed that the SER in each subcarrier is different due to the multi-band nonlinear distortion. Specifically, subcarriers in the middle and edge are less prone to symbol errors than the other subcarriers, which is also apparent from Fig. 3.4.

From the above discussion, it is quite obvious that by judicious subcarrier mapping and allocation, the performance of CA-MIMO-OFDM system in the presence of nonlinear HPA can be optimized.

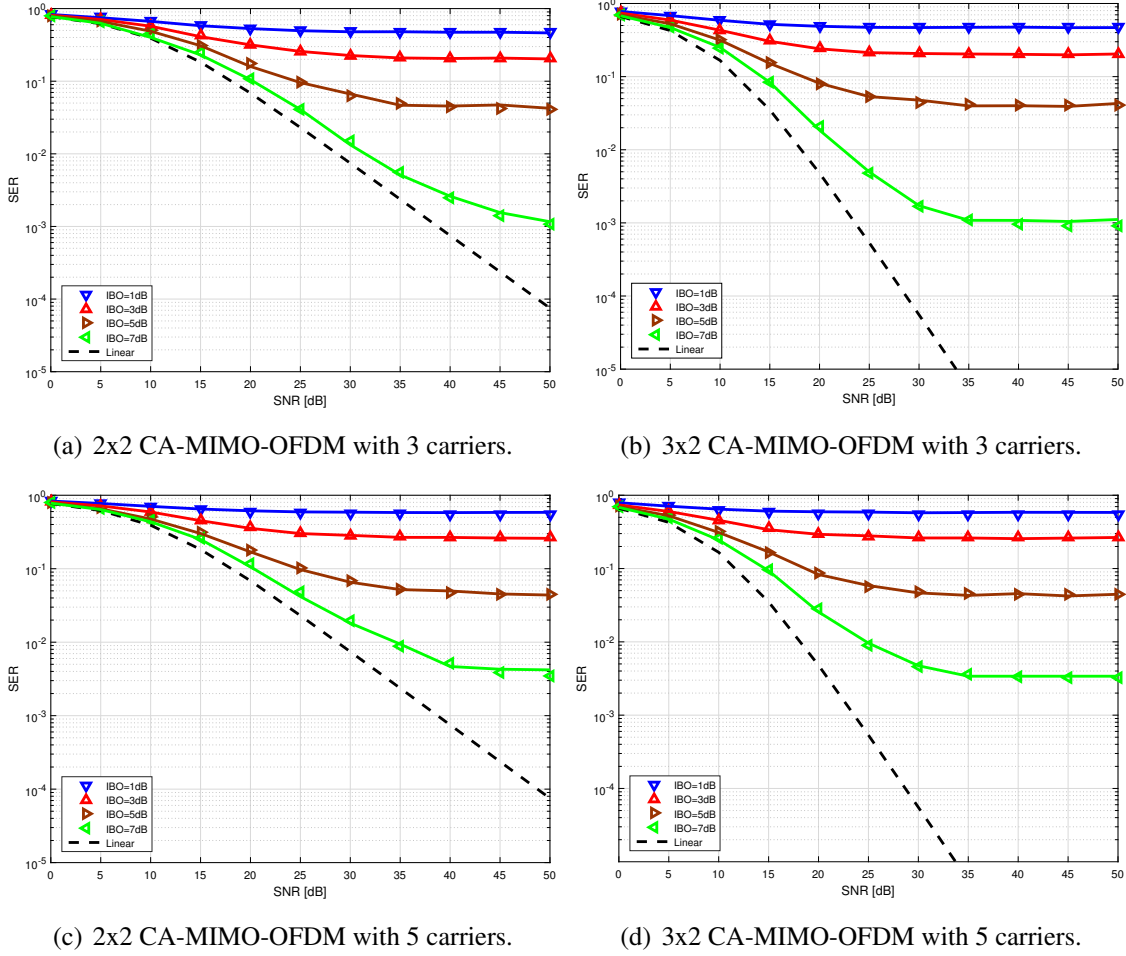


Figure 3.6: Analytical (solid lines) and simulation (marker points) results of average SER of different CA-MIMO-OFDM system in Rayleigh fading channel for 16-QAM, $N=2048$.

3.6.2 CA-MIMO-OFDM Simulation

In this section, we evaluate the symbol error performance of CA-MIMO-OFDM system impaired by nonlinear HPAs. The MIMO channel is characterized by the six delay taps with uniform power delay profile [68]. The number of subcarriers in each OFDM signal is $N = 2048$ with cyclic prefix of length $Ng = 512$. Each subcarrier contains symbols from a 16-QAM constellation. The simulation is averaged over 10^6 iterations, where for each iteration the data symbols and the channel are randomly and independently generated.

Fig. 3.6 shows the average SER performance as a function of AWGN SNR, i.e., P_{in}/N_o and IBO for different CA-MIMO-OFDM. As an ideal performance benchmark, system performance with linear HPA is also plotted. The impact of nonlinear distortion on the received signal decreases with increase in IBO. It can also be seen that the SER performance of CA-MIMO-

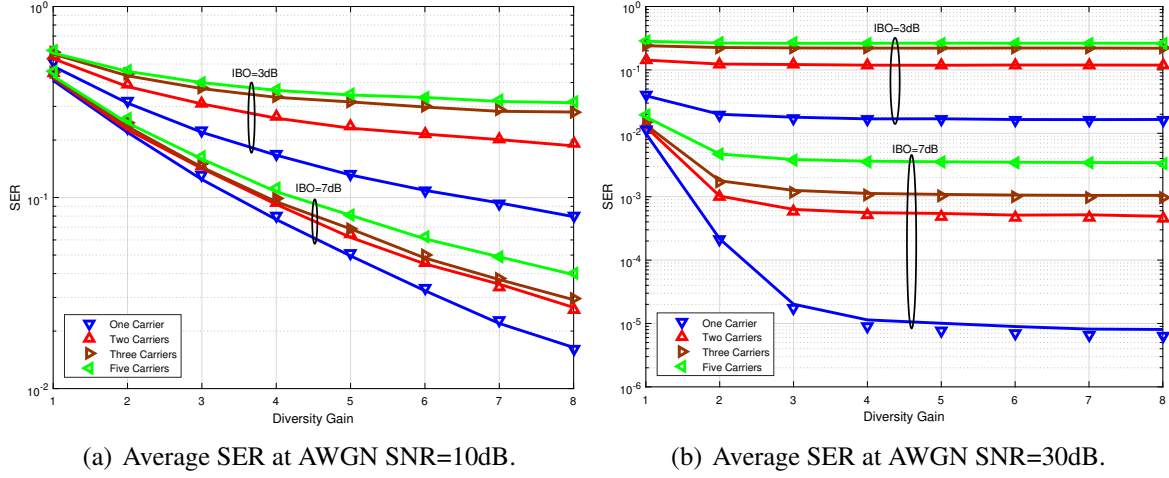


Figure 3.7: Analytical (solid lines) and simulation (marker points) results of average SER for different diversity gain of CA-MIMO-OFDM system in Rayleigh fading channel for 16-QAM, $N=2048$.

OFDM system improves with the increase in SNR up to some extent, after that increase in SNR has no significant impact on the SER performance. This is due to the fact that at high SNR, say $\text{SNR} > 40$ dB, the AWGN effect reduces and only nonlinear distortion caused by HPA affects the performance. Further, on increasing the value of IBO, the impact of nonlinear distortion can be reduced and the system performance of CA-MIMO-OFDM with nonlinear HPA approaches to the linear system. From Fig. 3.6(a) and 3.6(c), it can be observed that even for same IBO, there is considerable performance difference when the number of aggregated carriers is switched from three to five carriers in 2x2 CA-MIMO-OFDM system. This strengthens the fact that the increase in the number of aggregated carriers leads multi-band HPA to operate more towards nonlinear region which further degrades the error performance of CA-MIMO-OFDM system.

The impact of different diversity gain ($N_R - N_T + 1$) on the performance of CA-MIMO-OFDM systems is illustrated in Fig. 3.7. Although an increase in diversity gain improves the SER performance, however it is limited to the low SNR regime and high values of IBO as shown in Fig. 3.7(a). At higher SNR as shown in Fig. 3.7(b), there is no significant impact of diversity gain at low IBO, however SER performance improves with diversity gain (up to order of 4) at higher IBO.

It can also be inferred from the above figures that for low SNR, SER improves with the increase in AWGN SNR and diversity gain as AWGN and diversity outweigh the effects of nonlinearity, however the system performance is still lower than the linear system. In high SNR regime, the nonlinear noise dominates AWGN and diversity gain which leads to irreducible error floor. This justifies the discussion in Section 3.4.

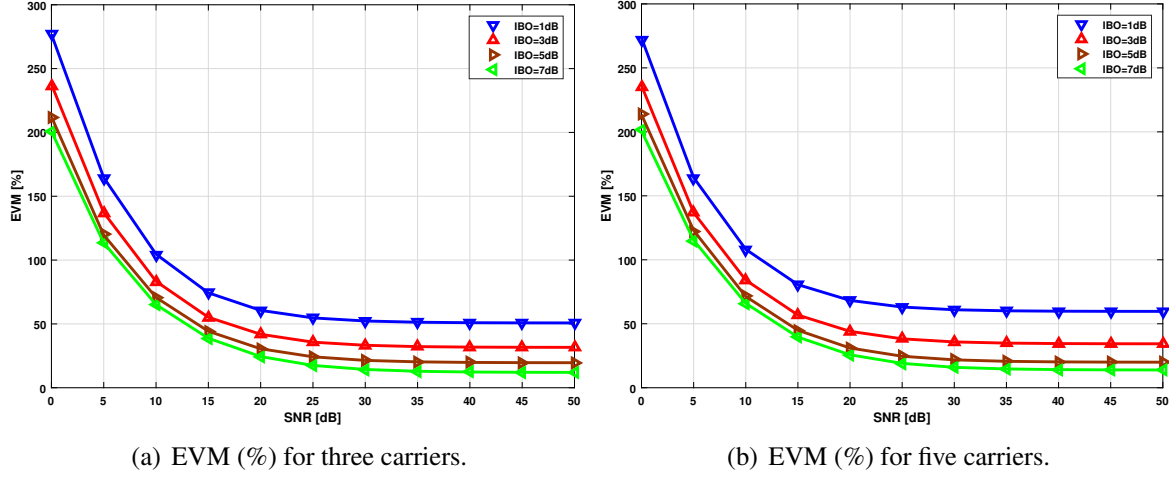


Figure 3.8: Analytical (solid lines) and simulation (marker points) results of average EVM for 2x2 CA-MIMO-OFDM system in Rayleigh fading channel for 16-QAM, N=2048.

Table 3.1: Transmit signal EVM (%) for 16-QAM CA-OFDM signal with N=2048.

Number of Carriers	One	Two	Three	Five
IBO=1dB	21.15	38.24	50.74	59.65
IBO=3dB	16.71	25.34	31.60	34.31
IBO=5dB	12.03	17.37	19.56	20.92
IBO=7dB	8.54	11.21	11.97	13.84

3.6.3 EVM Analysis

In this section, the EVM performance of a nonlinear CA-MIMO-OFDM system has been evaluated for the simulation setup defined in Section 3.6.2.

The transmit signal EVM can be defined as $EVM_b^{TX} = \frac{1}{N} \sum_{k=0}^{N-1} \sqrt{\frac{1}{SDR_{b,k}}}$. Table 3.1 provides useful insights on the effect of a different number of aggregated bands on the transmit signal EVM. It can be observed that EVM increases as the number of aggregated bands increases. On operating nonlinear HPA with higher back-off, EVM can be maintained within a tolerance level. As per 3GPP specifications TS 136.101 [111] and TS 136.104 [112], maximum transmit signal EVM for 16-QAM is 12.5%. It is quite obvious that higher IBO is required, if there are more number of aggregated bands, in order to keep the transmit signal EVM under 3GPP specification. This observation will be quite useful when estimating the power efficiency-energy consumption trade-off while designing a CA LTE-A network. More the aggregated bands better will be throughput, however it may reduce the power efficiency of the network. The impact of AWGN SNR on the average EVM at the receiver for different values of IBO for 2x2 CA-MIMO-OFDM system with three and five aggregated carriers is shown in Fig. 3.8.

Good agreement between analytical and simulation results of EVM at receiver validate the accuracy of the closed form expression of EVM derived in Section 3.5. In low SNR regime, the value of EVM is too high due to the composite effect of nonlinear noise, channel fading and AWGN. The increase in SNR reduces the impact of channel fading and AWGN which results in low EVM. However, the value of EVM at the receiver saturates to the transmit signal EVM in high SNR regime due to the nonlinear distortion caused by HPA. This concludes that generalized expression of EVM can be used to calculate EVM at both transmitter and receiver.

3.7 Conclusion

In this chapter, we investigated the performance of CA LTE-A system in the presence of nonlinear HPA for a multi-path Rayleigh fading MIMO channel. The nonlinear behavior of HPA has been modeled using the MB-GMP model. It has been shown that nonlinear distortions at the output of a HPA for the CA-MIMO-OFDM system can be canonically characterized in terms of complex attenuation factor and additive zero-mean Gaussian nonlinear noise. Closed-form generalized expressions of complex attenuation factor and nonlinear noise are derived that are applicable for any number of aggregated bands with any nonlinearity and memory depth. From the expressions, a complete analytical approach is presented to obtain the received SNR, SER and EVM for each subcarrier of the CA-MIMO-OFDM systems.

It was also observed that the nonlinear distortion caused by multi-band HPA increases with an increase in the number of aggregated bands which diminishes the gain due to diversity and SNR. Further, the nonlinear distortion impacts each subcarrier of the received CA-OFDM signal differently. The nonlinear distortions can be alleviated to a certain extent by increasing the IBO. Therefore, by judicious subcarrier mapping and appropriate selection of IBO, the performance of the CA-MIMO-OFDM system can be optimized in the presence of nonlinear HPA. The derived analytical results will be useful when estimating power efficiency and energy consumption trade-off while designing CA LTE-A based wireless systems without performing an extensive simulation or tedious experiments.

MU-MIMO-OFDM System with Nonlinear HPAs

This chapter investigates the impact of nonlinear HPAs on the MU-MIMO-OFDM system. The brief introduction and contributions related to above mentioned system are given in Section 4.1 followed by Section 4.2 which presents the system model. Section 4.3 and 4.4 describe the linear and nonlinear MU-MIMO-OFDM system with transmit preprocessing in the presence of multi-path Rayleigh fading channel, respectively. The mathematical expressions of SER and average capacity are derived in Section 4.5. The numerical and simulation results are presented in Section 4.6. Finally, Section 4.7 concludes the chapter.

4.1 Introduction

MU-MIMO-OFDM system consists of two complementary technologies: MU-MIMO and OFDM. The MU-MIMO serves the multiple users simultaneously in the same time-frequency resource which significantly increases the system capacity. On the other hand, OFDM provides robustness to ISI, high spectral efficiency and simple frequency-domain equalization. The merger of these two technologies significantly increases the spectral efficiency of the wireless communication system. To further enhance the spectral efficiency, CA can be used in MU-MIMO-OFDM system. However, CA-OFDM signals are highly sensitive to nonlinear distortion due to its high PAPR as stated in Chapter 3. Likewise, the CA MU-MIMO-OFDM system is also greatly affected by HPA nonlinearity. While the impact of linear MU-MIMO-OFDM system has been studied, we are not aware of any work that investigates the impact of transmitter nonlinearity on the performance of MU-MIMO OFDM system.

Motivated by above, this chapter investigates the performance of carrier-aggregated DB MU-MIMO-OFDM system in the presence of transmitter nonlinearity. The transmitter nonlinearity is modeled by a 2D-GMP model [113]. A transmit preprocessing technique as used in [31] is employed to mitigate the effects of inter-user interference. This technique assumes that perfect transmit channel state information (CSI) for all users is available at the BS. Further, it decomposes MU-MIMO channel into multiple parallel SU-MIMO channels. We show that the transmitter nonlinearity introduces an irreducible nonlinear interference in the DB MU-MIMO-OFDM system. Further, the nonlinear interference is a function of intermodulation and cross-modulation products from every user and increases as the number of users increases, thus significantly deteriorating the performance of the DB MU-MIMO-OFDM system. We also provide valuable insights on the impact of the aggregated carriers and number of users on the SER and overall capacity of the system. Some of the major contributions of the proposed work are summarized as follows:

- The mathematical expression of a nonlinear interference component is derived that is applicable for any number of users present in the nonlinear MU-MIMO-OFDM system. From the derived expressions, it is shown that the nonlinear interference can be decomposed into intra-user (nonlinear interference by an input signal of the desired user) and inter-user (nonlinear interference by input signals of other users) interference. Further, this nonlinear interference can not be eliminated by the conventional transmit preprocessing technique, and it increases with an increase in the number of users and number of aggregated bands.
- The mathematical expression for a SINR for both linear and nonlinear MU-MIMO-OFDM systems is derived and further utilized to obtain an analytical framework to evaluate the end-to-end performance of both systems in terms of SER and average capacity in the presence of multi-path Rayleigh fading channel.
- The conventional MU-MIMO systems are typically represented as $(N_T, [N_1, N_2, \dots, N_U])$, where N_T is the number of transmit antennas at BS and N_u is the number of receive antennas at each u^{th} user, where $u \in [1, U]$ and U is the total number of users. An additional dimension is introduced which is referred as the number of CCs. Hence, a CA MU-MIMO system is referred as $(B, N_T, [N_1, N_2, \dots, N_U])$, where $B \in [1, 2]$ denotes the number of aggregated CC. This additional dimension helps to evaluate the impact of B on the overall capacity of a MU-MIMO-OFDM system. Further, it also helps to measure the trade-offs between CA and transmitter nonlinearity in a MU-MIMO-OFDM system.

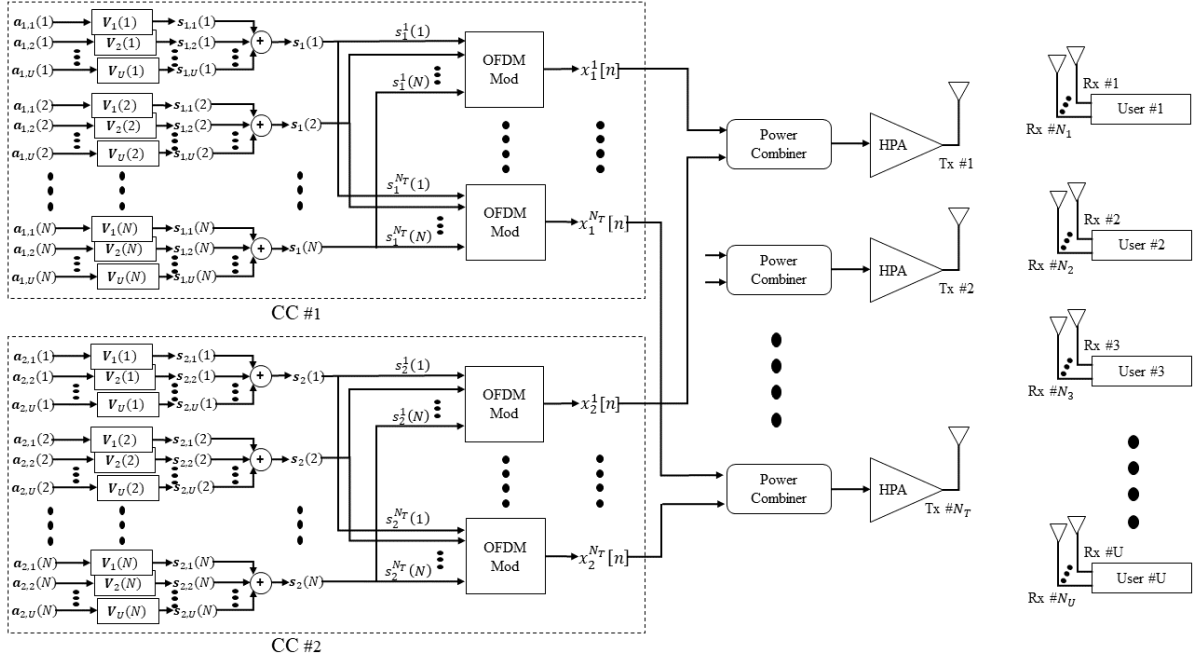


Figure 4.1: System Architecture of dual-band MU-MIMO-OFDM system.

- It is shown that due to transmitter nonlinearity there will be an irreducible error floor in the SER performance which increases with an increase in the number of users or number of aggregated CCs.
- It is shown that although the average capacity scales with B and U , the nonlinear interference severely limits the average capacity of a nonlinear MU-MIMO-OFDM system.

Simulation results are also reported to validate the theoretical analysis as well as to demonstrate the performance of the linear and nonlinear DB MU-MIMO-OFDM system in the presence of multi-path Rayleigh fading MU-MIMO channel. The obtained results are also compared with some special cases such as a conventional nonlinear SB MU-MIMO-OFDM [28], DB SU-MIMO-OFDM [113], and conventional MIMO-OFDM systems [70].

4.2 System Model

In this section, we present the system model for the downlink DB MU-MIMO-OFDM system. The system consists of N_T transmit antennas at the base station (BS) and U number of users each having N_u number of receive antennas as shown in Fig. 4.1. In DB MU-MIMO-OFDM system, BS simultaneously transmits $Q_u \times 1$ data symbol vector, $\mathbf{a}_{b,u}(k)$ at k^{th} subcarrier for

each u^{th} user in b^{th} CC, where Q_u is the number of parallel data streams and $b \in [1, 2]$. Denote B as the number of aggregated bands. It is worth noting, when $B = 1$, the aforementioned system reduces to SB MU-MIMO-OFDM system [28] and when $U = 1$, it reduces to a DB SU-MIMO-OFDM system [113]. For $Q_u = 1$, there will be no spatial multiplexing gain available at each user. For ease of exposition, the data symbols are assumed to be independent and identically distributed (i.i.d.) with zero mean and equal variance of P_u/B in each band. For every u^{th} user, $\mathbf{a}_{b,u}(k)$ is first precoded by $\mathbf{V}_u(k)$, a transmit precoding matrix to eliminate inter-user interference. $\mathbf{V}_u(k)$ is a $N_T \times Q_u$ matrix which transforms Q_u non-zero terms into N_T precoded symbols at output, i.e.,

$$\mathbf{s}_{b,u}(k) = \mathbf{V}_u(k) \mathbf{a}_{b,u}(k), \quad (4.1)$$

where $\mathbf{s}_{b,u}(k)$ is a $N_T \times 1$ precoded transmit vector at k^{th} subcarrier for each u^{th} user in b^{th} CC. The power of each element of the precoded transmit vector $\mathbf{s}_{b,u}(k)$ is P_{s_u} , which can be expressed as [70]:

$$P_{s_u} = \frac{Q_u P_u}{N_T B}. \quad (4.2)$$

Since, the data of all users are transmitted in the same time-frequency resource, thus the combined precoded data can be expressed as:

$$\mathbf{s}_b(k) = \sum_{u=1}^U \mathbf{s}_{b,u}(k). \quad (4.3)$$

These combined precoded vectors $\mathbf{s}_b(k)$ are then passed through OFDM modulators. The equivalent baseband expression of the OFDM signal with N subcarriers in b^{th} CC can be represented as [113]:

$$x_b^t[n] = \begin{cases} \frac{1}{N} \sum_{k=0}^{N-1} s_b^t(k) e^{j2\pi kn/N}, & -N_g \leq n \leq N-1 \\ 0, & \text{otherwise} \end{cases} \quad (4.4)$$

where N_g is length of cyclic prefix and n is the discrete time index. $s_b^t(k)$ is a complex data symbol in k^{th} subcarrier at t^{th} transmit antenna. By Central Limit Theorem [101], when N is large enough, $x_b^t[n]$ can be assumed to be complex Gaussian process with zero mean and variance $P_x = P_s/N$ [41], where P_s is the variance of $s_b^t(k)$ which can be given as:

$$P_s = \mathbb{E} \left[|s_b^t(k)|^2 \right]. \quad (4.5)$$

On substituting (4.3) in (4.5), we obtain

$$P_s = \sum_{u=1}^U \mathbb{E} \left[\left| s_{b,u}^t(k) \right|^2 \right] = \sum_{u=1}^U P_{s_u} = \sum_{u=1}^U \frac{Q_u P_u}{N_T B}. \quad (4.6)$$

As a consequence, the P_x can be rewritten as:

$$P_x = \frac{1}{N} \sum_{u=1}^U \frac{Q_u P_u}{N_T B}. \quad (4.7)$$

Therefore, the transmit power P_{in} can be written as:

$$P_{in} = \sum_{b=1}^B P_x = B P_x = \frac{1}{N} \sum_{u=1}^U \frac{Q_u}{N_T} P_u. \quad (4.8)$$

After OFDM modulation, the signals present in both CCs are aggregated together using power combiner [20] and then amplified it using HPA at each transmit antenna as shown in Fig. 4.1. For ease of exposition, we assumed that the channel between each user and BS is WSSUS multi-path Rayleigh fading MIMO channel, $\bar{\mathbf{H}}_u[n]$ which can be expressed as:

$$\bar{\mathbf{H}}_u[n] = \sum_{l=0}^{L-1} \bar{\mathbf{H}}_u^l \delta[n-l], \quad (4.9)$$

where L is total number of delay taps and $\bar{\mathbf{H}}_u^l$ is the MIMO channel response with dimension $N_u \times N_T$ for u^{th} user at l^{th} delay tap whose elements are i.i.d. $\mathcal{CN}(0, 1)$.

4.3 DB MU-MIMO-OFDM System with Linear HPAs

In this section, we evaluate the performance of a linear DB MU-MIMO-OFDM system in the presence of a multi-path Rayleigh fading channel. Without loss of generality, it is assumed that a linear HPAs with unit power gain is present at each transmit antenna of aforementioned system. The received vector for u^{th} user in b^{th} CC can be written as:

$$\bar{\mathbf{r}}_{b,u}[n] = \sum_{l=0}^{L-1} \bar{\mathbf{H}}_u^l \bar{\mathbf{y}}_b[n-l] + \bar{\mathbf{w}}_u[n], \quad (4.10)$$

where $\bar{\mathbf{r}}_{b,u}[n]$ is a $N_u \times 1$ received signal vector, $\bar{\mathbf{y}}_b[n]$ is a $N_T \times 1$ transmit signal vector whose elements are $y_b^t[n]$, and $\bar{\mathbf{w}}_u[n]$ is a $N_u \times 1$ noise vector, whose elements are i.i.d. $\mathcal{CN}(0, N_o)$. Since, we are considering a linear transmitter with unity gain, i.e., $\bar{\mathbf{y}}_b[n] = \bar{\mathbf{x}}_b[n]$, then (4.10)

can be rewritten as:

$$\bar{\mathbf{r}}_{b,u}[n] = \sum_{l=0}^{L-1} \bar{\mathbf{H}}_u^l \bar{\mathbf{x}}_b[n-l] + \bar{\mathbf{w}}_u[n]. \quad (4.11)$$

After the removal of cyclic prefix and taking the discrete Fourier transform (DFT) at receiver, the complex data symbols at k^{th} subcarrier for u^{th} user in b^{th} CC can be expressed as:

$$\mathbf{r}_{b,u}(k) = \mathbf{H}_u(k) \mathbf{s}_b(k) + \mathbf{w}_u(k), \quad (4.12)$$

where $\mathbf{H}_u(k) = \sum_{l=0}^{L-1} \bar{\mathbf{H}}_u^l e^{-j2\pi kl/N}$ is $N_u \times N_T$ effective MIMO channel matrix between BS and u^{th} user at k^{th} subcarrier, and $\mathbf{w}_u(k)$ is the noise vector at k^{th} subcarrier for u^{th} user. Using (4.1) and (4.3), the received vector $\mathbf{r}_{b,u}(k)$ in (4.12) can be rewritten as

$$\begin{aligned} \mathbf{r}_{b,u}(k) &= \mathbf{H}_u(k) \sum_{\hat{u}=1}^U \mathbf{V}_{\hat{u}}(k) \mathbf{a}_{b,\hat{u}}(k) + \mathbf{w}_u(k) \\ &= \underbrace{\mathbf{H}_u(k) \mathbf{V}_u(k) \mathbf{a}_{b,u}(k)}_{\text{I}} + \underbrace{\mathbf{H}_u(k) \sum_{\hat{u}=1, \hat{u} \neq u}^U \mathbf{V}_{\hat{u}}(k) \mathbf{a}_{b,\hat{u}}(k)}_{\text{II}} + \mathbf{w}_u(k), \end{aligned} \quad (4.13)$$

where the I term represents the desired signal for u^{th} user and the II term is the interference at u^{th} user from all the $U - 1$ users, which can be referred as inter-user interference. To mitigate this interference, a transmit precoding technique can be used which chooses $\mathbf{V}_u(k)$ nonzero precoding matrices for all U users at each subcarrier in such a manner that each user does not experience interference from the other $U - 1$ users [31], which can be defined as:

$$\mathbf{V}_u(k) = \mathbf{E}_u(k) \mathbf{F}_u(k), \quad (4.14)$$

where $\mathbf{F}_u(k)$ is a nonzero $A_u \times Q_u$ equivalent transmit precoding matrix and $\mathbf{E}_u(k)$ is a $N_T \times A_u$ orthonormal matrix whose columns are the nullspace of $\mathbf{\Theta}_u(k)$ [114] which can be computed as:

$$\text{SVD} \underbrace{\begin{bmatrix} \mathbf{H}_1(k) \\ \vdots \\ \mathbf{H}_{u-1}(k) \\ \mathbf{H}_{u+1}(k) \\ \vdots \\ \mathbf{H}_U(k) \end{bmatrix}}_{\mathbf{\Theta}_u(k)} = \mathbf{C}_u(k) \begin{bmatrix} \mathbf{\Sigma}_u(k) & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \hat{\mathbf{E}}_u(k)^H \\ \mathbf{E}_u(k)^H \end{bmatrix}, \quad (4.15)$$

where $\mathbf{C}_u(k)$ is a $N_T - A_u \times N_T - A_u$ unitary matrix, $\mathbf{\Sigma}_u(k)$ is a $N_u - A_u \times N_u - A_u$ square diagonal matrix with singular values on the diagonal, $\hat{\mathbf{E}}_u(k)$ is a $N_T \times N_T - A_u$ orthonormal matrix whose columns are the row space of $\mathbf{\Theta}_u(k)$ [114] and A_u can be defined as [31]:

$$A_u = N_T - \sum_{\hat{u}=1, \hat{u} \neq u}^U N_{\hat{u}}. \quad (4.16)$$

$\mathbf{E}_u(k)$ is used to eliminate inter-user interference, i.e., $\mathbf{H}_{\hat{u}}(k)\mathbf{E}_u(k) = \mathbf{0} \forall \hat{u} \in \{\{1, 2, \dots, U\} \setminus \{u\}\}$. Therefore on substituting (4.14), the received vector in (4.13) can be rewritten as:

$$\mathbf{r}_{b,u}(k) = \mathbf{H}_u(k)\mathbf{V}_u(k)\mathbf{a}_{b,u}(k) + \mathbf{w}_u(k) = \mathbf{H}_u(k)\mathbf{E}_u(k)\mathbf{F}_u(k)\mathbf{a}_{b,u}(k) + \mathbf{w}_u(k). \quad (4.17)$$

It can be observed from (4.17) that II term in (4.13) is eliminated and the DB MU-MIMO-OFDM system has been decomposed into U parallel DB SU-MIMO-OFDM systems. Denote $\hat{\mathbf{H}}_u(k) = \mathbf{H}_u(k)\mathbf{E}_u(k)$, (4.17) can be rewritten as:

$$\mathbf{r}_{b,u}(k) = \hat{\mathbf{H}}_u(k)\mathbf{F}_u(k)\mathbf{a}_{b,u}(k) + \mathbf{w}_u(k). \quad (4.18)$$

For every u^{th} user, $\hat{\mathbf{H}}_u(k)$ and $\mathbf{F}_u(k)$ is equivalent SU-MIMO-OFDM channel and corresponding equivalent transmit precoding matrix, respectively. The received signal (4.18) is processed by the $Q_u \times N_u$ decoding matrix $\mathbf{D}_u(k)$ and given as:

$$\hat{\mathbf{r}}_{b,u}(k) = \mathbf{D}_u(k)^H \mathbf{r}_{b,u}(k). \quad (4.19)$$

Since the equivalent SU-MIMO-OFDM channel $\hat{\mathbf{H}}_u(k)$ is known as the transmitter, the precoding and decoding matrix can be obtained from the SVD of the $\hat{\mathbf{H}}_u(k)$ which can be defined as:

$$\hat{\mathbf{H}}_u(k) = \mathbf{D}_u(k)\mathbf{\Xi}_u(k)\mathbf{F}_u(k)^H, \quad (4.20)$$

where $\mathbf{D}_u(k)$ and $\mathbf{F}_u(k)$ are the unitary matrices, and taking $N_{\max} = \max(N_u, A_u)$,

$$\mathbf{\Xi}_u(k) = \text{diag} \left[\sqrt{\lambda_u^{(1)}(k)}, \sqrt{\lambda_u^{(2)}(k)}, \dots, \sqrt{\lambda_u^{(Q_u)}(k)}, \underbrace{0, \dots, 0}_{N_{\max} - Q_u} \right], \quad (4.21)$$

where $\sqrt{\lambda_u^{(z)}(k)}$ are the eigenvalues of the complex Wishart matrix $\mathbf{W}_u(k)$ which is given as [115]:

$$\mathbf{W}_u(k) = \begin{cases} \hat{\mathbf{H}}_u(k)\hat{\mathbf{H}}_u(k)^H & \text{if } N_u \leq A_u \\ \hat{\mathbf{H}}_u(k)^H\hat{\mathbf{H}}_u(k) & \text{if } N_u > A_u \end{cases} \quad (4.22)$$

This method converts the MIMO channel into Q_u number of parallel SISO channel for each users. The value of the Q_u can be determined from the rank of $\hat{\mathbf{H}}_u(k)$ and can be defined as:

$$Q_u \leq \min_{k \in [1, K]} [\text{rank}(\hat{\mathbf{H}}_u(k))]. \quad (4.23)$$

On substituting (4.18) and (4.20) in (4.19), we get

$$\begin{aligned} \hat{\mathbf{r}}_{b,u}(k) &= \mathbf{D}_u(k)^H \mathbf{r}_{b,u}(k) = \mathbf{D}_u(k)^H (\mathbf{H}_u(k) \mathbf{E}_u(k) \mathbf{F}_u(k) \mathbf{a}_{b,u}(k) + \mathbf{w}_u(k)) \\ &= \mathbf{D}_u(k)^H \mathbf{D}_u(k) \mathbf{\Xi}_u(k) \mathbf{F}_u(k)^H \mathbf{F}_u(k) \mathbf{a}_{b,u}(k) + \mathbf{D}_u(k)^H \mathbf{w}_u(k) \\ &= \mathbf{\Xi}_u(k) \mathbf{a}_{b,u}(k) + \tilde{\mathbf{w}}_u(k) = \begin{bmatrix} \sqrt{\lambda_u^{(1)}(k)} a_{b,u}^{(1)}(k) \\ \sqrt{\lambda_u^{(2)}(k)} a_{b,u}^{(2)}(k) \\ \vdots \\ \sqrt{\lambda_u^{(Q_u)}(k)} a_{b,u}^{(Q_u)}(k) \end{bmatrix} + \begin{bmatrix} \tilde{w}_u^{(1)}(k) \\ \tilde{w}_u^{(2)}(k) \\ \vdots \\ \tilde{w}_u^{(Q_u)}(k) \end{bmatrix}. \end{aligned} \quad (4.24)$$

Since the elements of $\mathbf{w}_u(k)$ are i.i.d. $\mathcal{CN}(0, N_o)$ and $\mathbf{D}_u(k)$ is a unitary matrix, the elements of $\tilde{\mathbf{w}}_u(k)$ are also i.i.d.

4.4 DB MU-MIMO-OFDM System with Nonlinear HPAs

In this section, we analyze a dual-band MU-MIMO-OFDM system impaired with nonlinear HPAs at each transmit antenna. For ease of exposition, HPAs in all transmit antennas are assumed to have same behavioral characteristics [113]. The nonlinear HPA can be characterized by a 2D-GMP model which is given as [113]:

$$y_b^t[n] = \sum_{p=1}^P \sum_{q=1}^p \sum_{m=0}^{M-1} \sum_{g=-G}^G c_{p,q,m,g} x_b^t[n-m] |x_1^t[n-m-g]|^{p-q} |x_2^t[n-m-g]|^{q-1}, \quad (4.25)$$

where $y_b^t[n]$ is the output of nonlinear HPA corresponding to the OFDM signal $x_b^t[n]$ presented in b^{th} CC. $c_{p,q,m,g}$ is the nonlinearity coefficient of HPA, P is the nonlinearity order, G is the number of cross terms and M represents the memory depth [113]. The received complex data symbols at k^{th} subcarrier for u^{th} user in 1^{st} CC can be expressed as¹:

$$\mathbf{r}_{1,u}(k) = \mathbf{H}_u(k) \mathbf{y}_1(k) + \mathbf{w}_u(k), \quad (4.26)$$

¹Although the analysis have been done for 1^{st} CC, however the aforementioned analysis is also applicable to 2^{nd} CC as well.

where the t^{th} element of $\mathbf{y}_1(k)$ in (4.26) can be expressed as:

$$\begin{aligned} y_1^t(k) &= \sum_{n=0}^{N-1} y_1^t[n] e^{-j2\pi kn/N} \\ &= \sum_{n=0}^{N-1} \sum_{p=1}^P \sum_{q=1}^p \sum_{m=0}^{M-1} \sum_{g=-G}^G c_{p,q,m,g} x_1^t[n-m] |x_1^t[n-m-g]|^{p-q} |x_2^t[n-m-g]|^{q-1} e^{-j2\pi kn/N}. \end{aligned} \quad (4.27)$$

The received data signal in (4.27) can be canonically decomposed in terms of an attenuation factor, $\mu_1(k)$ multiplied with the desired signal, and an additive nonlinear interference, $\phi_1^t(k)$, which can be expressed as (refer Appendix A):

$$y_1^t(k) = \mu_1(k) s_1^t(k) + \phi_1^t(k), \quad (4.28)$$

where the attenuation factor $\mu_1(k)$ is considered as an unknown deterministic parameter and the additive nonlinear noise $\phi_1^t(k)$ is modeled as a zero-mean Gaussian process uncorrelated to the input signal (refer Appendix B) in order to make it mathematically tractable [11, 20]. Their expressions can be defined as:

$$\begin{aligned} \mu_1(k) &= \sum_{p=1}^P \sum_{q=1}^p \sum_{m=0}^{M-1} \left(c_{p,q,m,0} \Gamma \left(1 + \frac{p-q+2}{2} \right) + \sum_{g=-G, g \neq 0}^G c_{p,q,m,g} \Gamma \left(1 + \frac{p-q}{2} \right) \right) \\ &\quad \times \Gamma \left(1 + \frac{q-1}{2} \right) (P_{in})^{(p-1)/2} e^{-j2\pi km/N} \end{aligned} \quad (4.29)$$

and

$$\begin{aligned} \phi_1^t(k) &= \sum_{n=0}^{N-1} \sum_{p=1}^P \sum_{q=1}^p \sum_{m=0}^{M-1} \sum_{g=-G}^G c_{p,q,m,g} x_1^t[n-m] |x_1^t[n-m-g]|^{p-q} \\ &\quad \times |x_2^t[n-m-g]|^{q-1} e^{-j2\pi kn/N} - \mu_1(k) s_1^t(k). \end{aligned} \quad (4.30)$$

Note that $\mu_1(k)$ and $\phi_1^t(k)$ are function of number of users in nonlinear DB MU-MIMO-OFDM system. The $|x_1^t[\cdot]|^{p-q}$ and $|x_2^t[\cdot]|^{q-1}$ term in $\phi_1^t(k)$ are intermodulation and cross modulation

products, respectively, which can be expressed as:

$$|x_1^t[\cdot]|^{p-q} = \left| \underbrace{x_{1,u}^t[\cdot]}_{\text{I}} + \underbrace{\sum_{o=1, o \neq u}^U x_{1,o}^t[\cdot]}_{\text{II}} \right|^{p-q} \quad (4.31a)$$

$$|x_2^t[\cdot]|^{q-1} = \left| \underbrace{x_{2,u}^t[\cdot]}_{\text{I}} + \underbrace{\sum_{o=1, o \neq u}^U x_{2,o}^t[\cdot]}_{\text{II}} \right|^{q-1} \quad (4.31b)$$

The I term in (4.31a) and (4.31b) are the intermodulation and cross modulation products which appear due to the input signal of u^{th} user. It can be referred as intra-user nonlinear interference. However, the II term in (4.31a) and (4.31b) are the intermodulation and cross modulation products due to the input signals of remaining $U - 1$ users, which can be referred as inter-user nonlinear interference. This interference is different from the inter-user interference present in (4.13) which only depends on the linear part of the input signals of remaining $U - 1$ users. For instance, a linear system (with ideal HPAs) comprising of one BS and two users, the only interference term presents at user-1 is $\mathbf{H}_1(k)\mathbf{V}_2(k)\mathbf{a}_{b,2}(k)$ which can easily be filtered out using the transmit preprocessing technique. This interference is known as inter-user interference. However, in the presence of nonlinear HPAs, the total interference at user-1 is $\mathbf{H}_1(k)\mathbf{V}_2(k)\mathbf{a}_{b,2}(k) + \mathbf{H}_1(k)\Phi_1(k)$ where the first interference term can be filtered as in the case of linear HPAs. However, the second interference term cannot be eliminated using the conventional transmit preprocessing technique and it adds on to the desired signal².

For two users MIMO system, the intermodulation and cross modulation products in (4.31a) and (4.31b) can be expanded as $|x_{1,1}^t[\cdot] + x_{1,2}^t[\cdot]|^{p-q}$ and $|x_{2,1}^t[\cdot] + x_{2,2}^t[\cdot]|^{q-1}$, respectively. In case of SU system, $x_{1,2}^t[\cdot] = x_{2,2}^t[\cdot] = 0$ and the remaining terms $x_{1,1}^t[\cdot]$ and $x_{2,1}^t[\cdot]$ contributes to the nonlinear interference. Since, these nonlinear interferences are resultant of the desired signal, and are in absence of any other user, thus they can be referred as intra-user nonlinear interference. However, in the presence of additional users (for a MU system), other nonlinear interference terms are present which is referred here as inter-user nonlinear interference. They increase with increase in the number of users and they are more severe as they cannot be eliminated using the conventional transmit preprocessing technique. Thus, on increasing

²Please note that for massive MIMO systems it has been shown in [116] that the impact of hardware impairments at the BS vanishes asymptotically, however this is only true when $N \rightarrow \infty$ or the number of BS antennas grows very large.

the number of users, the inter-user nonlinear interference will also increase and significantly deteriorate the performance of the nonlinear DB MU-MIMO-OFDM system. From (4.26) and (4.28), the received vector for u^{th} user at k^{th} subcarrier can be expressed as:

$$\mathbf{r}_{1,u}(k) = \mathbf{H}_u(k)(\mu_1(k)\mathbf{s}_1(k) + \Phi_1(k)) + \mathbf{w}_u(k), \quad (4.32)$$

where $\Phi_1(k)$ is $N_t \times 1$ nonlinear interference vector whose elements are $\phi_1^t(k)$. On substituting (4.1), (4.3) (4.14) into (4.32), we obtain

$$\mathbf{r}_{1,u}(k) = \underbrace{\mu_1(k)\mathbf{H}_u(k)\mathbf{E}_u(k)\mathbf{F}_u(k)\mathbf{a}_{1,u}(k)}_{\text{Desired Signal}} + \underbrace{\mathbf{H}_u(k)\Phi_1(k)}_{\text{Nonlinear Interference}} + \underbrace{\mathbf{w}_u(k)}_{\text{AWGN}}. \quad (4.33)$$

It is easily noticeable from (4.30)-(4.31b) that $\Phi_1(k)$ depends on both intra-user and inter-user nonlinear interference, which can not be eliminated by using conventional transmit preprocessing technique. Due to this, the DB MU-MIMO-OFDM system with nonlinear HPA cannot be disintegrated into U parallel DB SU-MIMO-OFDM systems. Moreover, as the number of users increases, the inter-user nonlinear interference also increases which deteriorate the performance of nonlinear DB MU-MIMO-OFDM system. It is worth noting from (4.30) that for a nonlinear SB MU-MIMO-OFDM system ($x_2^n[n] = 0$), the nonlinear interference only depends on the intermodulation products rather than cross-modulation products, as a consequence, the error performance of the SB MU-MIMO-OFDM system is better than DB MU-MIMO-OFDM system in the presence of nonlinear HPAs. The received vector in (4.33) can be rewritten as:

$$\mathbf{r}_{1,u}(k) = \mu_1(k)\hat{\mathbf{H}}_u(k)\mathbf{F}_u(k)\mathbf{a}_{1,u}(k) + \mathbf{H}_u(k)\Phi_1(k) + \mathbf{w}_u(k). \quad (4.34)$$

The received signal (4.34) is decoded by $\mathbf{D}_u(k)$ which can be expressed as:

$$\hat{\mathbf{r}}_{1,u}(k) = \mathbf{D}_u(k)^H \mathbf{r}_{1,u}(k). \quad (4.35)$$

On substituting (4.20) and (4.34) in (4.35), we obtain

$$\hat{\mathbf{r}}_{1,u}(k) = \mu_1(k)\Xi_u(k)\mathbf{a}_{1,u}(k) + \mathbf{D}_u(k)^H \mathbf{H}_u(k)\Phi_1(k) + \mathbf{D}_u(k)^H \mathbf{w}_u(k). \quad (4.36)$$

On substituting $\Phi_1(k) = \mathbf{V}_u(k)\tilde{\Phi}_1(k)$, where $\tilde{\Phi}_1(k)$ is a $Q_u \times 1$ precoded nonlinear distortion vector whose elements are $\tilde{\phi}_1^z(k) \sim \mathcal{CN}(0, N\sigma_{\phi_1}^2)$ (refer Appendix D), (4.36) can be modified

as:

$$\hat{\mathbf{r}}_{1,u}(k) = \mu_1(k) \mathbf{\Xi}_u(k) \mathbf{a}_{1,u}(k) + \mathbf{D}_u(k)^H \mathbf{H}_u(k) \mathbf{V}_u(k) \tilde{\mathbf{\Phi}}_1(k) + \mathbf{D}_u(k)^H \mathbf{w}_u(k) \quad (4.37)$$

Using (4.14) and (4.20), we rewrite (4.37) as:

$$\begin{aligned} \hat{\mathbf{r}}_{1,u}(k) &= \mu_1(k) \mathbf{\Xi}_u(k) \mathbf{a}_{1,u}(k) + \mathbf{\Xi}_u(k) \tilde{\mathbf{\Phi}}_1(k) + \tilde{\mathbf{w}}_u(k) \\ &= \mu_1(k) \begin{bmatrix} \sqrt{\lambda_u^{(1)}(k)} a_{1,u}^{(1)}(k) \\ \sqrt{\lambda_u^{(2)}(k)} a_{1,u}^{(2)}(k) \\ \vdots \\ \sqrt{\lambda_u^{(Q_u)}(k)} a_{1,u}^{(Q_u)}(k) \end{bmatrix} + \begin{bmatrix} \sqrt{\lambda_u^{(1)}(k)} \tilde{\phi}_1^{(1)}(k) \\ \sqrt{\lambda_u^{(2)}(k)} \tilde{\phi}_1^{(2)}(k) \\ \vdots \\ \sqrt{\lambda_u^{(Q_u)}(k)} \tilde{\phi}_1^{(Q_u)}(k) \end{bmatrix} + \begin{bmatrix} \tilde{w}_u^{(1)}(k) \\ \tilde{w}_u^{(2)}(k) \\ \vdots \\ \tilde{w}_u^{(Q_u)}(k) \end{bmatrix}. \end{aligned} \quad (4.38)$$

4.5 End-to-End Performance Analysis

In this section, we will derive the analytical expression of the SER and capacity of both linear and nonlinear dual-band MU-MIMO-OFDM system³. Please note that these derivations are mainly based on the probability distribution function (PDF) of the eigenvalues of the complex Wishart matrix defined in (4.22). We begin with the derivation of the SER and capacity of the linear dual-band MU-MIMO-OFDM system, which is followed by the derivation of SER and capacity of the nonlinear dual-band MU-MIMO-OFDM system.

4.5.1 Linear dual-band MU-MIMO-OFDM system

The SNR per subcarrier of the z^{th} channel for u^{th} user at b^{th} CC can be defined as:

$$SNR_{b,u}^{(z)}(k) = \frac{\lambda_u^{(z)}(k) P_u}{N_o B}. \quad (4.39)$$

The SER per subcarrier of the data symbol for M-QAM dual-band MU-MIMO-OFDM system for u^{th} user at b^{th} CC can be expressed as [70]:

$$SER_{b,u}^{(z)}(k) \approx 4 \left(1 - \frac{1}{\sqrt{M}} \right) Q \left(\sqrt{\frac{3 \lambda_u^{(z)}(k) P_u}{(M-1) N_o B}} \right), \quad (4.40)$$

³Since a massive MIMO is an extension of MU-MIMO concept with large number of antennas [117, 118], thus all the derived expressions can also be utilized to evaluate the performance of the massive MIMO systems. However, in the proposed work we have restricted our results to MU-MIMO systems.

where $Q(\cdot)$ is a Q function [105]. The average SER across all Q_u channels can be given as:

$$SER_{b,u}(k) \approx 4 \left(1 - \frac{1}{\sqrt{M}}\right) \mathbb{E}_{\lambda_u(k)} \left[Q \left(\sqrt{\frac{3\lambda_u(k)P_u}{(M-1)N_oB}} \right) \right], \quad (4.41)$$

where $\mathbb{E}_{\lambda_u(k)} [\cdot]$ is the expectation operator conditioned on $\lambda_u(k)$ that can be randomly selected from $\lambda_u^{(z)}(k)$ whose PDF can be given as [119]:

$$f_{\lambda_u(k)}(\lambda_u(k)) = \frac{1}{Q_u} \sum_{z=0}^{Q_u-1} \sum_{\gamma=0}^z \sum_{\xi=0}^{2\gamma} \frac{(-1)^\xi (2\gamma)!}{2^{2z-\xi} \gamma! \xi! (\Delta + \gamma)!} \binom{2z-2\gamma}{z-\gamma} \binom{2\gamma+2\Delta}{2\gamma-\xi} \lambda_u^{\Delta+\xi}(k) e^{-\lambda_u(k)}. \quad (4.42)$$

Thus, the SER in (4.41) can be further evaluated as:

$$SER_{b,u}(k) \approx 4 \left(1 - \frac{1}{\sqrt{M}}\right) \frac{1}{Q_u} \sum_{z=0}^{Q_u-1} \sum_{\gamma=0}^z \sum_{\xi=0}^{2\gamma} \frac{(-1)^\xi (2\gamma)!}{2^{2z-\xi} \gamma! \xi! (\Delta + \gamma)!} \binom{2z-2\gamma}{z-\gamma} \binom{2\gamma+2\Delta}{2\gamma-\xi} I, \quad (4.43)$$

where $I = \int_0^\infty Q\left(\sqrt{\lambda_u(k)A}\right) \lambda_u^{\Delta+\xi}(k) e^{-\lambda_u(k)} d\lambda_u(k)$ and $A = \frac{3P_u}{(M-1)N_oB}$. Using (3.321.1) of [105], I in (4.43) can be expanded as:

$$I = \int_0^\infty \left(\frac{1}{2\pi} \int_{\sqrt{\lambda_u(k)A}}^\infty e^{-\frac{x^2}{2}} dx \right) \lambda_u^{\Delta+\xi}(k) e^{-\lambda_u(k)} d\lambda_u(k). \quad (4.44)$$

On changing the order of integration, (4.44) converts to

$$I = \int_0^\infty \frac{1}{2\pi} e^{-\frac{x^2}{2}} \left(\int_0^{\frac{x^2}{A}} \lambda_u^{\Delta+\xi}(k) e^{-\lambda_u(k)} d\lambda_u(k) \right) dx. \quad (4.45)$$

Now using (3.351.1) and (3.461.2) of [105], (4.45) can be expressed as:

$$I = \frac{(\Delta + \xi)!}{2} \left\{ 1 - \sum_{d=0}^{\Delta+\xi} \binom{2d}{d} \frac{\sqrt{A}}{2^d (A+2)^{d+1/2}} \right\}. \quad (4.46)$$

On substituting (4.46) in (4.43), the closed form expression of the SER per subcarrier for M-QAM of the dual-band MU-MIMO-OFDM system can be expressed as:

$$SER_{b,u}(k) \approx \frac{1}{Q_u} \left(1 - \frac{1}{\sqrt{M}}\right) \sum_{z=0}^{Q_u-1} \sum_{\gamma=0}^z \sum_{\xi=0}^{2\gamma} \frac{(-1)^\xi (2\gamma)! (\Delta + \xi)!}{2^{2z-\xi-1} \gamma! \xi! (\Delta + \gamma)!} \binom{2z-2\gamma}{z-\gamma} \\ \times \binom{2\gamma+2\Delta}{2\gamma-\xi} \left\{ 1 - \sum_{d=0}^{\Delta+\xi} \binom{2d}{d} \frac{\sqrt{A}}{2^d (A+2)^{d+1/2}} \right\}. \quad (4.47)$$

Finally, the SER across all subcarriers can be evaluated as:

$$SER_{b,u} = \frac{1}{N} \sum_{k=0}^{N-1} SER_{b,u}(k). \quad (4.48)$$

The overall capacity of DB MU-MIMO-OFDM system can be evaluated as:

$$C = \frac{1}{N} \sum_{b=1}^B \sum_{u=1}^U \sum_{k=0}^{N-1} \mathbb{E} [\log_2 (1 + A_o \lambda_u(k))], \quad (4.49)$$

where $A_o = \frac{P_u}{N_o B}$. Using (4.42), the capacity in (4.49) can be further evaluated as:

$$C = \frac{1}{N Q_u} \sum_{b=1}^B \sum_{u=1}^U \sum_{k=0}^{N-1} \sum_{z=0}^{Q_u-1} \sum_{\gamma=0}^z \sum_{\xi=0}^{2\gamma} \frac{(-1)^\xi (2\gamma)!}{2^{2z-\xi} \gamma! \xi! (\Delta + \gamma)!} \binom{2z-2\gamma}{z-\gamma} \binom{2\gamma+2\Delta}{2\gamma-\xi} \\ \times \int_0^\infty \log_2 (1 + A_o \lambda_u(k)) \lambda_u^{\Delta+\xi}(k) e^{-\lambda_u(k)} d\lambda_u(k). \quad (4.50)$$

Using (4.337.5) of [105], the closed form expression of overall capacity of the linear dual-band MU-MIMO-OFDM system can be expressed as:

$$C = \frac{1}{\ln(2) Q_u} \sum_{b=1}^B \sum_{u=1}^U \sum_{z=0}^{Q_u-1} \sum_{\gamma=0}^z \sum_{\xi=0}^{2\gamma} \frac{(-1)^\xi (2\gamma)!}{2^{2z-\xi} \gamma! \xi! (\Delta + \gamma)!} \binom{2z-2\gamma}{z-\gamma} \binom{2\gamma+2\Delta}{2\gamma-\xi} \\ \times \sum_{\varepsilon=0}^{\Delta+\xi} \frac{(\Delta + \xi)!}{(\Delta + \xi - \varepsilon)!} \left\{ \frac{(-1)^{\Delta+\xi-\varepsilon-1}}{A_o^{\Delta+\xi-\varepsilon}} e^{\frac{1}{A_o}} \text{Ei} \left(-\frac{1}{A_o} \right) + \sum_{\kappa=1}^{\Delta+\xi-\varepsilon} (\kappa-1)! \left(-\frac{1}{A_o} \right)^{\Delta+\xi-\varepsilon-\kappa} \right\}, \quad (4.51)$$

where $\text{Ei}[\cdot]$ is an exponential integral [105].

4.5.2 Nonlinear dual-band MU-MIMO-OFDM system

The nonlinear dual-band MU-MIMO-OFDM system suffers from the nonlinear interference due to the presence of nonlinear HPAs in each transmitting branch of the BS. In this section, we evaluate the SINR per subcarrier and then derive the SER and system capacity. The SINR per subcarrier of the z^{th} channel for u^{th} user in first CC can be defined as:

$$SINR_{1,u}^{(z)}(k) = \frac{\lambda_u^{(z)}(k) |\mu_1(k)|^2 \frac{P_u}{B}}{P_{int}^{(z)}(k) + N_o}. \quad (4.52)$$

where $P_{int}^{(z)}(k)$ is the power of the interference component of (4.38) which can be expressed as:

$$P_{int}^{(z)}(k) = \lambda_u^{(z)}(k) E \left[\left| \tilde{\phi}_1^{(z)}(k) \right|^2 \right] = \lambda_u^{(z)}(k) E \left[\left| \sum_{t=1}^{N_T} V_u^{(z,t)*}(k) \phi_1^t(k) \right|^2 \right], \quad (4.53)$$

where $V_u^{(z,t)*}(k)$ is the t^{th} element of the z^{th} eigen vector taken from the matrix $\mathbf{V}_u(k)$. The interference power in (4.53) can be evaluated as [70]:

$$\begin{aligned} P_{int}^{(z)}(k) &= \lambda_u^{(z)}(k) E \left[\sum_{t=1}^{N_T} \left| V_u^{(z,t)*}(k) \right|^2 \left| \phi_1^t(k) \right|^2 \right] \\ &= \lambda_u^{(z)}(k) \sum_{t=1}^{N_T} \left| V_u^{(z,t)*}(k) \right|^2 E \left[\left| \phi_1^t(k) \right|^2 \right] \\ &= \lambda_u^{(z)}(k) \underbrace{\sigma_{\phi_1(k)}^2 \sum_{t=1}^{N_T} \left| V_u^{(z,t)*}(k) \right|^2}_{=1, \forall z,k,u} = \lambda_u^{(z)}(k) \sigma_{\phi_1(k)}^2, \end{aligned} \quad (4.54)$$

where $\sigma_{\phi_1(k)}^2 = E \left[\left| \phi_1^t(k) \right|^2 \right]$ is the variance of the nonlinear interference which is derived in Appendix C. On putting (4.54) in (4.52), the SINR per subcarrier can be expressed as:

$$SINR_{1,u}^{(z)}(k) = \frac{\lambda_u^{(z)}(k) |\mu_1(k)|^2 \frac{P_u}{B}}{\lambda_u^{(z)}(k) \sigma_{\phi_1(k)}^2 + N_o}. \quad (4.55)$$

Therefore, the average SER for u^{th} user in first CC can be expressed as:

$$\begin{aligned} SER_{1,u} \approx & 4 \left(1 - \frac{1}{\sqrt{M}} \right) \frac{1}{Q_u N} \sum_{k=0}^{N-1} \sum_{z=0}^{Q_u-1} \sum_{\gamma=0}^z \sum_{\xi=0}^{2\gamma} \frac{(-1)^\xi (2\gamma)!}{2^{2z-\xi} \gamma! \xi! (\Delta + \gamma)!} \binom{2z-2\gamma}{z-\gamma} \\ & \times \left(\frac{2\gamma+2\Delta}{2\gamma-\xi} \right) \int_0^\infty Q \left(\sqrt{\frac{\lambda_u(k) |\mu_1(k)|^2 \frac{3P_u}{(M-1)B}}{\lambda_u(k) \sigma_{\phi_1(k)}^2 + N_o}} \right) \lambda_u^{\Delta+\xi}(k) e^{-\lambda_u(k)} d\lambda_u(k) \quad (4.56) \end{aligned}$$

and the overall capacity of the nonlinear dual-band MU-MIMO-OFDM system can be expressed as:

$$\begin{aligned} C = & \frac{1}{NQ_u} \sum_{b=1}^B \sum_{u=1}^U \sum_{k=0}^{N-1} \sum_{z=0}^{Q_u-1} \sum_{\gamma=0}^z \sum_{\xi=0}^{2\gamma} \frac{(-1)^\xi (2\gamma)!}{2^{2z-\xi} \gamma! \xi! (\Delta + \gamma)!} \binom{2z-2\gamma}{z-\gamma} \left(\frac{2\gamma+2\Delta}{2\gamma-\xi} \right) \\ & \times \int_0^\infty \log_2 \left(1 + \frac{\lambda_u(k) |\mu_1(k)|^2 \frac{P_u}{B}}{\lambda_u(k) \sigma_{\phi_1(k)}^2 + N_o} \right) \lambda_u^{\Delta+\xi}(k) e^{-\lambda_u(k)} d\lambda_u(k). \quad (4.57) \end{aligned}$$

The analytical expressions (4.56) and (4.57) can be calculated using numerical methods⁴. They provides the complete analysis of DB MU-MIMO-OFDM system in the presence of nonlinear HPA in terms of SER and system capacity over the multi-path Rayleigh fading channel⁵.

4.6 Simulation Results and Discussion

In this section, we simulate the performance of dual-band MU-MIMO-OFDM system impaired with nonlinear HPA. The 2D-GMP model with $P = 3$, $M = 4$, and $G = 2$ is used to model the nonlinear behavior of HPA. The saturation power of the HPA is given as $P_{sat} = P_T / N_T$, where P_T is the total power of the aforementioned system [70]. The nonlinear distortion can be reduced by operating HPA with IBO, which is defined as $IBO = P_{sat} / P_{in}$. Consequently, at higher IBO, HPA moves away from the saturation region and operates more towards the linear region. Different sets of coefficients for different IBOs are used in the 2D-GMP model to characterize the nonlinear HPA behavior [113].

The MU-MIMO-OFDM system with N_T transmit antenna, U users and N_u receive antenna at each user, is denoted as $(B, N_T, [N_1, N_2, \dots, N_U])$. In the present context, $B = 1$ corresponds to a single-band system whereas $B = 2$ represents the dual-band system. We assume that all

⁴Several software are available to compute the expressions given in (4.56) and (4.57) such as MATLAB, Mathematica and MAPLE. Throughout this dissertation, the ‘integral’ function of MATLAB is used to evaluate this kind of expression.

⁵As mentioned Chapter-3, the closed-form expression for SER is derived for high SNR values but not applicable for low SNR values. The SER is calculated numerically using (3.24) for low SNR values.

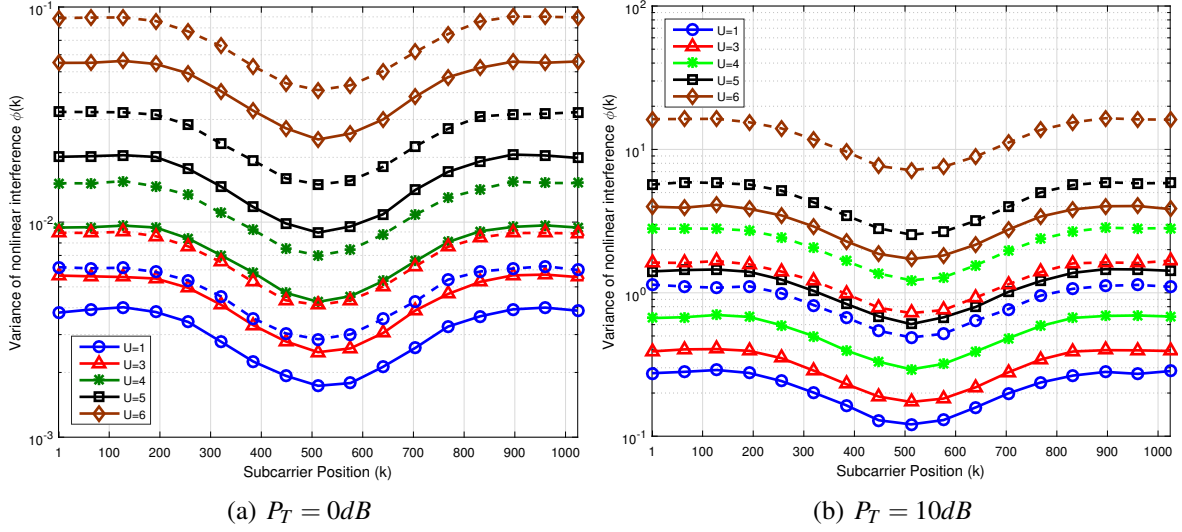


Figure 4.2: Solid lines and dotted lines represent the variance of nonlinear interference $\phi(k)$ at each subcarrier of SB and DB MU-MIMO-OFDM system, respectively, with $IBO = 5dB$.

users have same number of receive antennas, i.e., $N_u = N_R \forall u \in [1, U]$ and BS transmits Q number of data streams to each user ($Q_1 = Q_2 = \dots = Q_U = Q$). In addition, we also assume that $N_T > (U - 1)N_R$ for the existence of nonzero precoding matrices at each transmitter. Similarly, a SU-MIMO-OFDM system with N_T transmit and N_R receive antennas is denoted as (B, N_T, N_R) . As a performance benchmark, we also simulate the performance of aforementioned systems in the presence of unity gain linear HPA⁶. The number of OFDM subcarriers is taken to be $N = 1024$ and length of the cyclic prefix $N_g = 256$. As mentioned before, the channel between the BS and each user is modeled as a multi-path Rayleigh fading MIMO channel with six delay taps and with the uniform power delay profile [28].

4.6.1 Nonlinear Interference

The variance of the nonlinear interference $\phi(k)$ caused by HPA at each subcarrier in both single and dual-band MU-MIMO-OFDM systems with a total transmit power $P_T = 0dB$ and $P_T = 10dB$ are shown in Fig. 4.2(a) and Fig. 4.2(b), respectively. It can be observed that the nonlinear interference increases when we switch from the single band to the dual-band system. This can be explained as follows. The cross modulation products caused due to interference from the input signal present in the other CC in the DB system results in additional nonlinear interference as compared to single band system where nonlinear interference is only

⁶In case of linear HPA, both multi-user and single-user system have the same SER performance for each user [28].

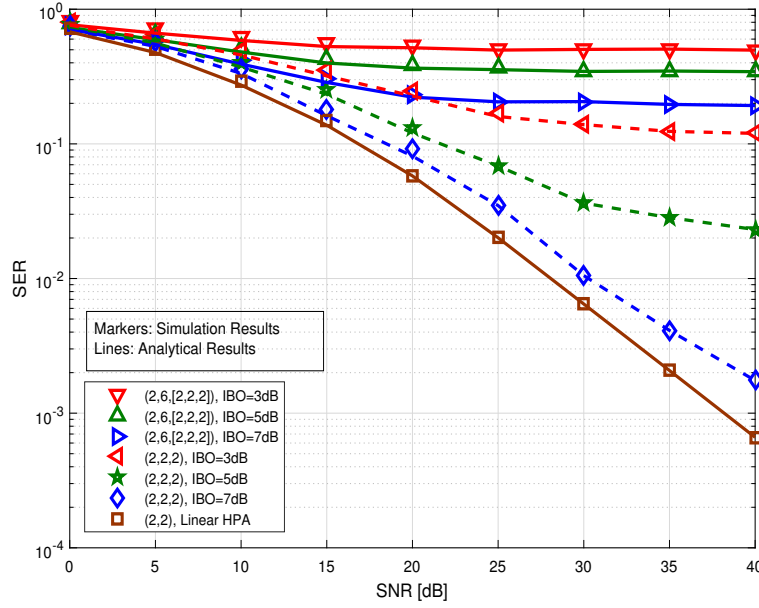


Figure 4.3: Performance comparison in terms of average SER of dual-band MU-MIMO-OFDM system and SU-MIMO-OFDM system for different IBOs.

caused by intermodulation products. It is also worth noting that the nonlinear interference increases significantly on increasing the number of users which can also be deduced from (4.30)-(4.31b). The higher number of users in the system causes excessive intermodulation and cross-modulation products. It results in intra and inter-user nonlinear interference which can not be eliminated by conventional transmit preprocessing technique. This supports the discussion in Section 4.4. Further, these distortions become more severe with the increase in total power supply P_T . As observed from Fig. 4.2, when the value of P_T rises from 0dB to 10dB, the variance of nonlinear interference increases by approximately 100 times in DB-MU-MIMO-OFDM system with six users.

From the above discussion, it is quite obvious that the nonlinear interference depends on the number of users, the number of aggregated bands and total transmit power. Further, the conventional precoding schemes which alleviate the inter-user interference in linear MU-MIMO-OFDM systems are ineffective for the nonlinear MU-MIMO-OFDM systems.

4.6.2 SER Performance

Fig. 4.3 shows the average SER comparison between (2,6,[2,2,2]) MU-MIMO-OFDM and (2,2,2) SU-MIMO-OFDM system in presence of nonlinear HPA for different signal-to-noise-ratio (SNR), i.e., P_T/N_o . Two data streams per user, i.e., $Q = 2$ are transmitted in both systems. Excellent agreement between the simulation and the theoretical results can be observed, which

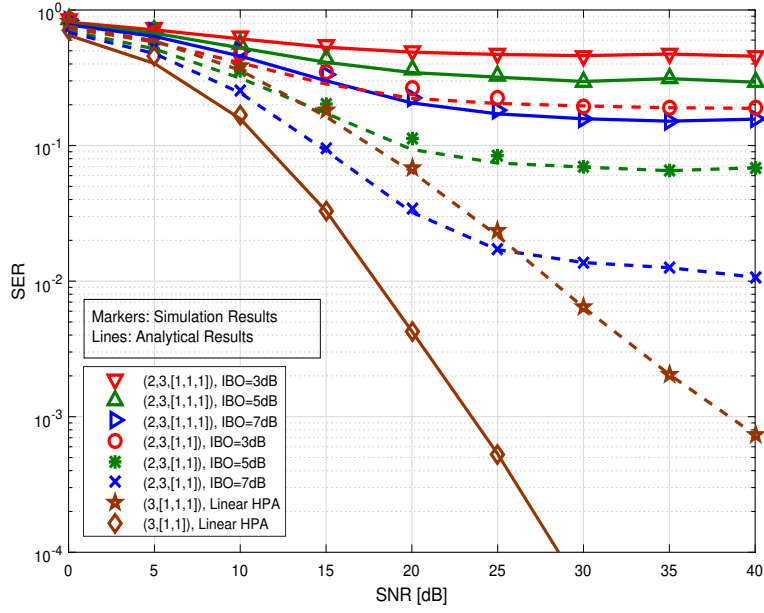


Figure 4.4: Performance comparison in terms of average SER of two different dual-band MU-MISO-OFDM systems for different IBOs.

supports the analytical framework proposed in the chapter. It is also obvious that the SER performance of both systems improves with an increase in IBO because at higher IBO HPA tends to operate closer to the linear region. However, it is important to note that for the same IBO, there is a considerable difference in the SER performance of SU and MU systems. This is due to the fact that the transmitted MU-MIMO OFDM signal at each antenna is the summation of signals of three users thus resulting in very high PAPR. High PAPR shifts the operating point of HPA more towards saturation region which causes significant nonlinear distortion. As mentioned earlier, this nonlinear distortion cannot be eliminated using conventional transmit preprocessing technique. Hence, the dual-band MU-MIMO system has worse SER performance as compared to the corresponding dual-band SU-MIMO system in the presence of nonlinear HPAs. This is also consistent with the discussion in Section 4.4.

The impact of the number of users on the performance of the multi-user system in the presence of nonlinear HPA is shown in Fig. 4.4. Here, two dual-band multi-user multi-input-single-output (MU-MISO) OFDM systems, $(2,3,[1,1,1])$ and $(2,3,[1,1])$ are considered with $Q = 1$. It can be deduced from Fig. 4.4 that when U increases from 2 to 3, the error performance of nonlinear DB MU systems degrades. This is because of loss in transmit diversity per user and increase in PAPR of the transmitted signal which forces HPA to operate in more saturation region which leads to severe nonlinear distortion. Moreover, nonlinear interference being a function of U increases with U , which further deteriorate the SER performance of the system.

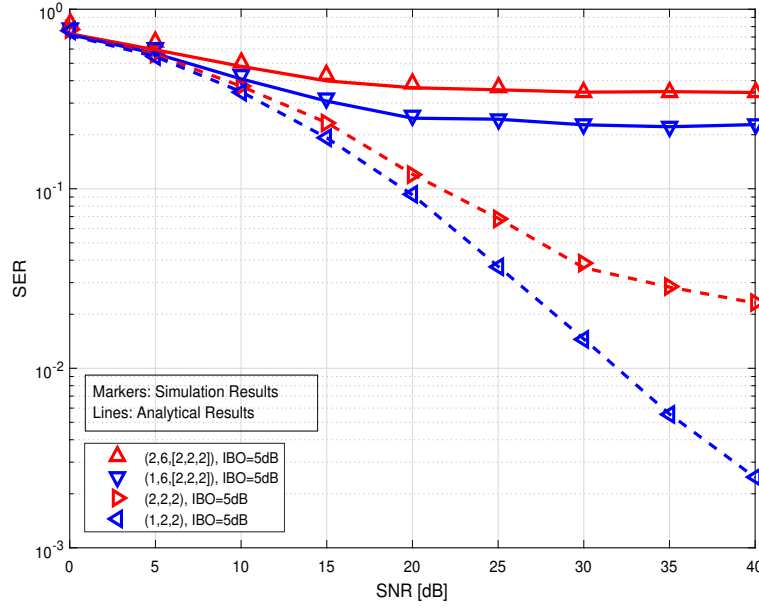


Figure 4.5: Performance comparison in terms of average SER of different MIMO-OFDM systems with $IBO = 5dB$.

Fig. 4.5 illustrates the joint impact of the number of users and number of aggregated bands on different nonlinear MU-MIMO-OFDM systems. It can be seen that there is a substantial degradation in SER performance when we shift from SB system to DB system in either SU and MU configurations. However, this degradation becomes more critical in the MU system due to the presence of inter-user nonlinear interference, which can also be inferred from (4.30). Thus, increasing the number of users or number of aggregated bands aggravates the SER performance of nonlinear MU-MIMO-OFDM system.

4.6.3 Capacity Analysis

Throughout this section, it is assumed that BS is equipped with $N_T = 15$ transmit antennas and each user has $N_R = 2$ receive antennas with noise variance $N_o = 0dB$. The transmit power allocated for each user is $P_u = P_T/U \forall u \in [1, U]$.

Fig. 4.6(a) and Fig. 4.6(b) illustrates the overall capacity of SB and DB MU-MIMO-OFDM systems, respectively, for different values of U and P_T . The results for capacity of the linear SB MU-MIMO-OFDM system is in line with the results obtained in [31], which validates the aforementioned analysis. Good agreement between the analytical and simulation results of the capacity verifies the mathematical expressions obtained in (4.51) and (4.57). It is quite obvious that the overall capacity of a DB system is higher than the capacity of the SB system. This can also be observed through dotted lines in Fig. 4.6.

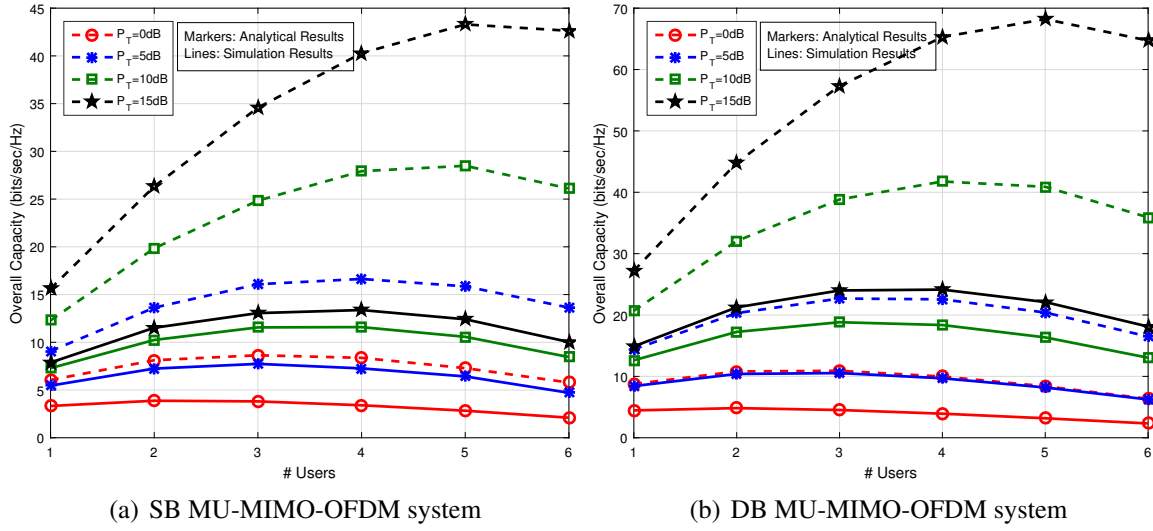


Figure 4.6: Dotted lines and solid lines represent overall system capacity with linear and nonlinear HPA, respectively. The results are plotted with respect to U and different values of total transmit power P_T for $\text{IBO}=5\text{dB}$.

Table 4.1: Overall capacity (bits/sec/Hz) of different CA MU-MIMO-OFDM system with $P_T = 15\text{dB}$ and $\text{IBO}=5\text{dB}$.

CA MU-MIMO-OFDM System	Linear System		Nonlinear System	
	B=1	B=2	B=1	B=2
(B,15,2)	15.61	27.21	7.88	14.89
(B,15,[2,2,2,2,2])	40.28	65.29	13.88	24.14
(B,15,[2,2,2,2,2,2])	43.3	68.21	12.41	22.11

Further, the capacity of the system may increase with the increase in the number of users. For instance, when BS is accommodating $U = 5$ users with the total transmit power of $P_T = 15\text{dB}$, the capacity of the system is about five-fold the capacity of the single user system which can also be inferred from Table 4.1. This is due to the transmit preprocessing technique at the BS which segregates the $(15,[2,2,2,2,2])$ system into five $(7,2)$ system [31]. Further increase in the number of users (i.e., $U > 5$) leads to a reduction in the capacity of the system because of the trade-off between transmit diversity and elimination of inter-user interference.

For the case of nonlinear MU-MIMO-OFDM system (represented by solid lines), the capacity is significantly reduced due to the nonlinear distortion caused by HPAs, which is also evident from Fig. 4.6. Further, the increase in the number of users also affects the overall capacity of the system because higher the number of users leads to more inter-user nonlinear interference which reduces the capacity of the system. For instance, the overall capacity of $(2,15,[2,2,2,2,2])$ system is 68.21 bits/sec/Hz in the presence of linear transmitter which reduces

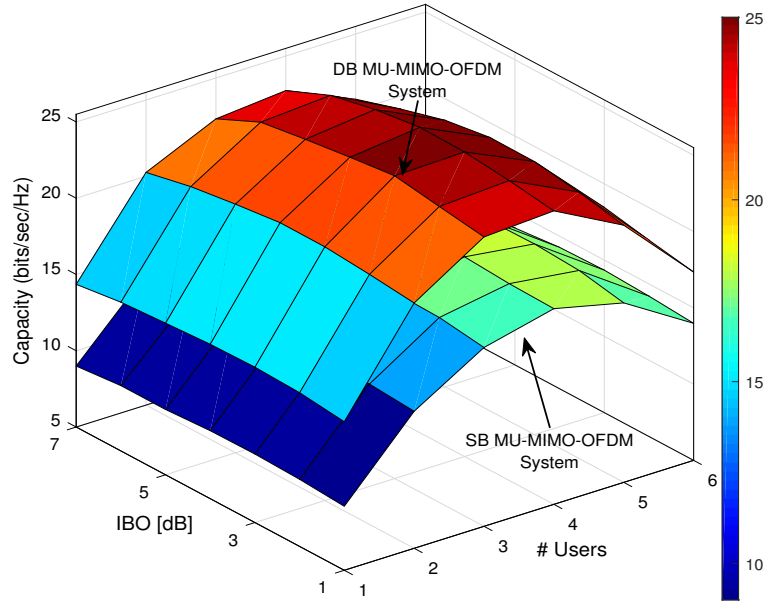


Figure 4.7: Overall capacity of the nonlinear SB (lower curve) and DB (upper curve) MU-MIMO-OFDM system with different number of users and at different IBOs with $P_T = 15dB$.

to 22.11 bits/sec/Hz with nonlinear transmitter when $P_T = 15dB$ as shown in Table 4.1. From Table 4.1, it can also be observed that the nonlinear MU-MIMO-OFDM system attains a maximum capacity of 24.14 bits/sec/Hz with four users whereas linear MU-MIMO-OFDM system attains a maximum capacity of 68.21 bits/sec/Hz with five users even though the total transmit power and number of transmit antennas are same for both the systems.

Fig. 4.7 shows the impact of IBOs and U on the capacity of the nonlinear MU-MIMO-OFDM system. As discussed before, at higher IBO HPA is operating more towards linear region which reduces the nonlinear distortion and correspondingly increases the SDNR of the system. As a consequence, the capacity also increases. For fixed saturation power (P_{sat}), increase in IBO reduces P_{in} , thus limiting the transmission power of the system, which in turn may adversely affect the SDNR. For instance, when with $U = 4$, the capacity of nonlinear DB MU-MIMO-OFDM system increases from 24.24 bits/sec/Hz at IBO=1dB to 25.02 bits/sec/Hz at IBO=4dB, however, it reduces to 22.73 bits/sec/Hz at IBO=7dB. Similar behavior can be noticed for different values of U . It can also be noticed from Fig. 4.7 that an optimum capacity can be achieved for particular IBO and U . This holds for both SB and DB systems.

4.7 Conclusion

This chapter evaluated the impact of the nonlinear transmitter on the performance of the CA MU-MIMO-OFDM system in the presence of multi-path Rayleigh fading channel. It was

shown that the transmit preprocessing technique which is conventionally used to mitigate the inter-user interference in a linear MU-MIMO-OFDM system is ineffective for the nonlinear MU-MIMO-OFDM system. The analytical expressions for nonlinear interference caused due to transmitter nonlinearity and SINR are derived. It was observed that the nonlinear interference consists of two parts: intra-user nonlinear interference which is caused by the input signal of the desired user and inter-user nonlinear interference caused by the input signals of the other users. Both inter-user and intra-user nonlinear interference were shown to be function of number of users and number of aggregated bands. It was also observed that the nonlinear interference deteriorates the performance of the nonlinear DB MU-MIMO-OFDM system by introducing an irreducible error floor earlier, thus limiting its gain vis-a-vis a DB SU-MIMO system, SB MU-MIMO system, and conventional MIMO system.

The overall capacity of the DB MU-MIMO-OFDM system was also evaluated in the presence of both linear and nonlinear transmitters. For the case of the linear transmitter, the capacity scales with the number of users and number of aggregated bands. Hence, a linear DB MU-MIMO-OFDM system significantly outperforms other linear MIMO-OFDM systems. However, this trend is unapparent in the presence of nonlinear transmitters. Although for nonlinear MU-MIMO-OFDM system capacity scales moderately with the number of users and bands however the gains are severely limited when compared to the linear system. Thus, the transmitter nonlinearity not only compromises the reliability but also restricts the average capacity of the system. Hence, it is imperative to take into account the impact of nonlinear transmitters while evaluating the performance of MU-MIMO-OFDM system. Further, a new transmit preprocessing technique should be evolved to mitigate this nonlinear interference, which may be the subject of future work.

Joint Impact of Nonlinear HPA and Non-ideal Oscillators

In the last two chapters, the impact of HPA nonlinearity has been investigated on MIMO-OFDM based systems. However, another hardware impairment such as phase noise has been overlooked so far. This chapter evaluates the performance of MU-MIMO-OFDM system in the presence of both HPA nonlinearity and phase noise. The brief introduction and contributions related to the above mentioned system are given in Section 5.1 followed by Section 5.2 which provides a brief overview of the linear MU-MIMO-OFDM system as well as discusses the behavioral model for HPA and phase noise. Section 5.3 describes the nonlinear MU-MIMO-OFDM system impaired with phase noise. Section 5.4 derives the closed-form expression of SER and system capacity. The simulation results are presented in Section 5.5. Finally, Section 5.6 concludes the chapter.

5.1 Introduction

Phase noise is a random phase distortion in the OFDM subcarriers due to the random phase variation of the local oscillators present at the transmitter and receiver. It significantly degrades the efficiency and performance of the system [88]. Some of the researchers have investigated the joint effects of phase noise and HPA nonlinearity on OFDM-based systems [96–99]. However, most of the prior work on the analytical evaluation of the joint impact of phase noise and transmitter nonlinearity has been restricted to OFDM systems. Since MU-MIMO-OFDM systems are fundamentally different from conventional OFDM systems, we believe the joint impact of the above impairments will shed interesting insights into the performance of such systems, thus motivating the proposed work.

In this chapter, we present a theoretical framework to study the joint impact of non-ideal oscillators and nonlinear HPAs on the downlink MU-MIMO-OFDM system. In the proposed framework, it is presumed that the BS employs a transmit preprocessing technique to mitigate the effects of MUI [31]. This technique virtually decouples the MU-MIMO channel into several independent SU-MIMO channel. Furthermore, the SU-MIMO channel decomposes into multiple parallel streams. It is shown that the received data symbol affected by phase noise and HPA nonlinearity can be canonically decomposed into following parts, namely 1) the desired signal multiplied by an attenuation factor and CPE, 2) ICI, and 3) additive nonlinear distortion noise which is uncorrelated with the desired signal. The main contributions of the chapter are summarized as follows:

- The mathematical expression of an instantaneous SDINR for MU-MIMO-OFDM system impaired with phase noise and nonlinear HPA is derived and then used to obtain the closed-form expression for both SER and overall capacity.
- The derived closed-form expressions can be used for any nonlinear MU-MIMO-OFDM system irrespective of the fact whether the phase noise is being generated at the transmitter or receiver or both. Furthermore, it is also proven that the variance of nonlinear distortion noise and AWGN is independent of phase noise.
- It is shown that the phase noise characteristic is independent of the number of users. Thus, the error performance of the MU-MIMO-OFDM system in the presence of the phase noise remains same irrespective of the number of users.
- It is shown that both nonlinear distortion and phase noise introduce an irreducible error floor. This reduces the system reliability and also affects the system capacity.

5.2 System Model

In this section, we present the MU-MIMO-OFDM system that exploits the benefits of two complementary technologies: MU-MIMO and OFDM. MU-MIMO provides diversity as well as spatial multiplexing gains whereas OFDM assists in mitigating the ISI. The frequency domain analysis for linear MU-MIMO-OFDM systems (with ideal HPAs and oscillators) is simple and straightforward thanks to the orthogonality between the OFDM subcarriers as well as precoding employed in MU-MIMO to eliminate MUI. However, nonlinear HPAs introduce intermodulation which degrades the signal quality whereas non-ideal oscillators cause phase noise which disrupts the orthogonality among the subcarriers. These impairments make the frequency domain analysis more complex, however it is realizable as shown later in this chapter.

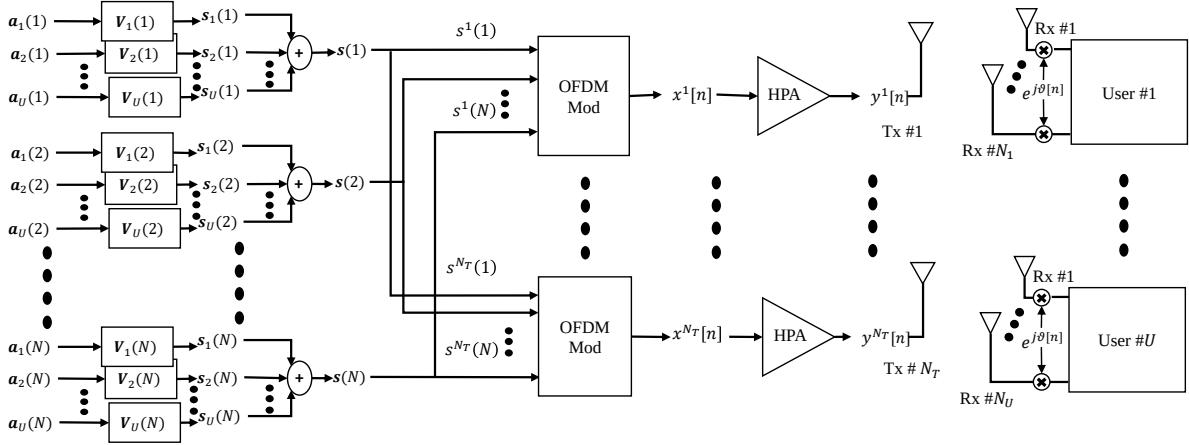


Figure 5.1: System Architecture of Downlink MU-MIMO-OFDM system.

In the following subsections, we provide a brief overview of a linear MU-MIMO-OFDM system as well as a behavioral model used for HPA and phase noise.

5.2.1 Linear MU-MIMO-OFDM System

A downlink MU-MIMO-OFDM system consists of N_T transmit antennas at the BS, U number of users each having N_u number of receive antennas as shown in Fig 5.1. BS precodes the $\mathbf{a}_u(k)$ data symbol vector with $\mathbf{V}_u(k)$ precoded matrix to eliminate MUI at each subcarrier. The $\mathbf{s}^t(k)$ transmit precoded data vector of all users is then converted to time domain signal through OFDM modulator. The complex baseband expression of the OFDM signal $x^t[n]$ at t^{th} transmit antenna can be expressed as:

$$x^t[n] = \frac{1}{N} \sum_{k=0}^{N-1} s^t(k) e^{j2\pi kn/N}. \quad (5.1)$$

By CLT, $x^t[n] \sim \mathcal{CN}(0, P_{in})$ for large N , where P_{in} can be given as [120]:

$$P_{in} = \frac{1}{N} \sum_{u=1}^U \frac{Q_u}{N_T} P_u. \quad (5.2)$$

After OFDM modulation, the time domain signals are up-converted and then passed through HPAs at each transmitting chain. The channel between BS and each user is assumed to be the WSSUS Rayleigh fading channel [120]. At the receiver, $\bar{\mathbf{r}}_u[n]$ is a $N_u \times 1$ received

down-converted discrete-time domain OFDM signal vector at u^{th} user which can be written as:

$$\bar{\mathbf{r}}_u[n] = \sum_{l=0}^{L-1} \bar{\mathbf{H}}_u^l \bar{\mathbf{y}}[n-l] + \bar{\mathbf{w}}_u[n], \quad (5.3)$$

where $\bar{\mathbf{H}}_u^l$ is the $N_u \times N_T$ channel matrix at l^{th} delay tap for u^{th} user, L is the total number of delay taps, $\bar{\mathbf{y}}[n]$ is a $N_T \times 1$ transmit signal vector whose elements are $y^t[n] = x^t[n]O(|x^t[n]|)$ where $O(\cdot)$ is the gain of the HPA, and $\bar{\mathbf{w}}_u[n]$ is a $N_u \times 1$ noise vector, whose elements are i.i.d. $\mathcal{CN}(0, N_o)$. Without loss of generality, we assume HPA with unit gain which implies $y^t[n] = x^t[n]$, thus (5.3) can be rewritten as:

$$\bar{\mathbf{r}}_u[n] = \sum_{l=0}^{L-1} \bar{\mathbf{H}}_u^l \bar{\mathbf{x}}[n-l] + \bar{\mathbf{w}}_u[n]. \quad (5.4)$$

After the OFDM demodulation, the complex data symbols at k^{th} subcarrier of u^{th} user can be expressed as:

$$\mathbf{r}_u(k) = \mathbf{H}_u(k)\mathbf{s}(k) + \mathbf{w}_u(k), \quad (5.5)$$

where $\mathbf{H}_u(k)$ is $N_u \times N_T$ effective MIMO channel matrix between BS and u^{th} user at k^{th} subcarrier and can be computed by taking the DFT of $\bar{\mathbf{H}}_u^l$ with all taps, and $\mathbf{w}_u(k)$ is the noise vector at k^{th} subcarrier for u^{th} user. Similar to (4.13), the received vector $\mathbf{r}_u(k)$ in (5.5) can be rewritten as:

$$\begin{aligned} \mathbf{r}_u(k) &= \mathbf{H}_u(k) \sum_{\hat{u}=1}^U \mathbf{V}_{\hat{u}}(k) \mathbf{a}_{\hat{u}}(k) + \mathbf{w}_u(k) \\ &= \underbrace{\mathbf{H}_u(k) \mathbf{V}_u(k) \mathbf{a}_u(k)}_{\text{Desired Signal}} + \underbrace{\mathbf{H}_u(k) \sum_{\hat{u}=1, \hat{u} \neq u}^U \mathbf{V}_{\hat{u}}(k) \mathbf{a}_{\hat{u}}(k)}_{\text{MUI}} + \mathbf{w}_u(k). \end{aligned} \quad (5.6)$$

As mentioned earlier to eliminate MUI, a transmit precoding matrix $\mathbf{V}_u(k)$ for all U users at each subcarrier is used [31], which can be defined as:

$$\mathbf{V}_u(k) = \mathbf{E}_u(k) \mathbf{F}_u(k), \quad (5.7)$$

where $\mathbf{F}_u(k)$ is a $A_u \times Q_u$ equivalent transmit precoding matrix to provide spatial multiplexing gain and $\mathbf{E}_u(k)$ is a $N_T \times A_u$ precoding matrix to eliminate MUI, i.e., $\mathbf{H}_{\hat{u}}(k) \mathbf{E}_u(k) = \mathbf{0} \forall \hat{u} \in \{\{1, 2, \dots, U\} \setminus \{u\}\}$. The computation of $\mathbf{E}_u(k)$ and $\mathbf{F}_u(k)$ are given in Section 4.3. Similar

to (4.24), the decoded received vector $\hat{\mathbf{r}}_u(k)$ can be written as:

$$\hat{\mathbf{r}}_u(k) = \mathbf{\Xi}_u(k) \mathbf{a}_u(k) + \tilde{\mathbf{w}}_u(k) = \begin{bmatrix} \sqrt{\lambda_u^{(1)}(k)} a_u^{(1)}(k) \\ \sqrt{\lambda_u^{(2)}(k)} a_u^{(2)}(k) \\ \vdots \\ \sqrt{\lambda_u^{(Q_u)}(k)} a_u^{(Q_u)}(k) \end{bmatrix} + \begin{bmatrix} \tilde{w}_u^{(1)}(k) \\ \tilde{w}_u^{(2)}(k) \\ \vdots \\ \tilde{w}_u^{(Q_u)}(k) \end{bmatrix}, \quad (5.8)$$

where $\mathbf{\Xi}_u(k)$ is a singular matrix whose elements $\lambda_u^{(z)}(k)$ are the singular value at z^{th} data stream for u^{th} user.

5.2.2 HPA Model

In this section, we describe the HPA model at each t^{th} transmit antenna of BS. It is assumed that the HPA in all transmitting chains have same input-output characteristics, thus we drop the superscript t for ease of readability. The output of the nonlinear HPA can be expressed as [121]:

$$y[n] = \Lambda(x[n]) e^{j(\arg(x[n]) + \theta(x[n]))}, \quad (5.9)$$

where $\Lambda(\cdot)$ and $\theta(\cdot)$ are the AM/AM and AM/PM distortion. In this chapter, we consider the HPAs at the transmitter to be solid-state power amplifier (SSPA). The behavioral characteristics of SSPA can be modeled using a Rapp model¹ [39]. Thus, $\Lambda(\cdot)$ and $\theta(\cdot)$ can be expressed as:

$$\Lambda(x[n]) = \frac{|x[n]|}{\left[1 + \left(\frac{|x[n]|}{A_{sat}}\right)^{2v}\right]^{\frac{1}{2v}}}, \quad \theta(x[n]) = \alpha_\theta \left(\frac{|x[n]|}{A_{sat}}\right)^4, \quad (5.10)$$

where A_{sat} is an input saturation amplitude of HPA, v is the smoothness parameter that regulates the changeover from linear to saturation region and α_θ is typically set to zero [96]. Therefore, using (5.10), (5.9) can be rewritten as:

$$y[n] = \frac{|x[n]| e^{j(\arg(x[n]))}}{\left[1 + \left(\frac{|x[n]|}{A_{sat}}\right)^{2v}\right]^{\frac{1}{2v}}} = \frac{x[n]}{\left[1 + \left(\frac{|x[n]|}{A_{sat}}\right)^{2v}\right]^{\frac{1}{2v}}} = x[n] O(|x[n]|), \quad (5.11)$$

¹ Although the results have been shown using Rapp model, the proposed analytical framework is applicable for other models as well such as Saleh Model [40], polynomial model [41], etc.

where $O(|x[n]|)$ is the gain of the HPA and can be given as:

$$O(|x[n]|) = \left[1 + \left(\frac{|x[n]|}{A_{sat}} \right)^{2v} \right]^{-\frac{1}{2v}}. \quad (5.12)$$

Using the Bussgang theorem [121], the output of the nonlinear HPA can be decomposed into a attenuation factor μ which attenuates the input signal and additive Gaussian nonlinear distortion $\phi[n] \sim \mathcal{CN}(0, \sigma_\phi^2)$ which is uncorrelated with the input signal. Thus, (5.11) can be expressed as:

$$y[n] = \mu x[n] + \phi[n]. \quad (5.13)$$

The attenuation factor μ can be derived by setting $v = 1$ in (5.11) as suggested in [122]:

$$\mu = \frac{\mathbb{E}[y[n]x[n]^\dagger]}{\mathbb{E}[|x[n]|^2]} = \frac{A_{sat}}{2\sigma_{in}} \left[\frac{2A_{sat}}{\sigma_{in}} - \sqrt{\pi} \operatorname{erfc} \left(\frac{A_{sat}}{\sigma_{in}} \right) e^{\frac{A_{sat}^2}{P_{in}}} \left(\frac{2A_{sat}^2}{P_{in}} - 1 \right) \right]. \quad (5.14)$$

Similarly, the variance of additive Gaussian nonlinear distortion $\phi[n]$ can be computed as [122]:

$$\sigma_\phi^2 = \mathbb{E}[|y[n]|^2] - \mu^2 \mathbb{E}[|x[n]|^2] = P_{in} \left[\frac{A_{sat}^2}{P_{in}} \left(1 + \frac{A_{sat}^2}{P_{in}} e^{\frac{A_{sat}^2}{P_{in}}} \operatorname{Ei} \left(\frac{A_{sat}^2}{P_{in}} \right) \right) - \mu^2 \right], \quad (5.15)$$

The nonlinear distortion can be suppressed by operating HPA with high input back-off (IBO), which is defined as $\text{IBO} = A_{sat}^2/P_{in}$. Thus, on increasing the IBO, HPA will operate more towards linear region rather than saturation region. As a consequence, the nonlinear distortion will reduce and the output of the HPA will become linear. However, it can be observed from (5.15) that the variance of nonlinear noise also depends on the power of the input signal P_{in} which further depends on the number of users as evident from (5.2). Consequently, the variance of nonlinear distortion depends on the number of users in the system and increases with increase in the number of users which significantly deteriorates the performance of the nonlinear MU-MIMO-OFDM system.

5.2.3 Phase Noise Model

Several mathematical models have been proposed in the literature to characterize the behavior of the phase noise. In our work, we model phase noise as Gaussian random variable $\vartheta[n] \sim \mathcal{N}(0, \sigma_\vartheta^2)$ to analyze the effects of phase noise on MU-MIMO system [89, 98, 99]. The statistics of this model are well defined in [89]. The variance of phase noise σ_ϑ^2 can easily be

calculated from oscillator parameters as [98]:

$$\sigma_{\vartheta}^2 = \frac{2K_{\vartheta}}{\pi} \tan^{-1} \left(\frac{B_w}{B_{3\text{dB}}} \right) + \frac{B_w N_f}{\pi}, \quad (5.16)$$

where K_{ϑ} is the phase noise factor, $B_{3\text{dB}}$ is the half-power bandwidth, B_w is the signal bandwidth and N_f is the superimposed 35 dB noise floor [98]. However, for ease of simplicity, our model is parametrized by only the phase noise variance². It may be noted that the transmit signal will be same if the phase noise is generated before or after the HPA.

5.3 MU-MIMO-OFDM System with Nonlinear HPAs and Phase Noise

In this section, we characterize the receive signal at each u^{th} user in MU-MIMO-OFDM system impaired by nonlinear HPA and phase noise. Therefore, the received OFDM signal vector for u^{th} user after down-conversion can be defined as:

$$\bar{\mathbf{r}}_u[n] = \left(\sum_{l=0}^{L-1} \bar{\mathbf{H}}_u^l \bar{\mathbf{y}}[n-l] + \bar{\mathbf{w}}_u[n] \right) e^{j\vartheta[n]}. \quad (5.17)$$

On substituting (5.13) in (5.17), we obtain

$$\bar{\mathbf{r}}_u[n] = \left(\sum_{l=0}^{L-1} \bar{\mathbf{H}}_u^l (\mu \bar{\mathbf{x}}[n-l] + \bar{\Phi}[n-l]) + \bar{\mathbf{w}}_u[n] \right) e^{j\vartheta[n]}, \quad (5.18)$$

where $\bar{\Phi}[n]$ is a $N_T \times 1$ nonlinear distortion vector whose elements are $\phi^l[n]$. After taking DFT of (5.18) and using (5.1), the received complex symbols at k^{th} subcarrier of u^{th} user can be

²It is important to note that this analysis is valid irrespective of the choice of phase noise model if the statistics of the phase noise process are known accurately.

expressed as:

$$\begin{aligned}
\mathbf{r}_u(k) &= \frac{1}{N} \sum_{n=0}^{N-1} \sum_{k'=0}^{N-1} (\mathbf{H}_u(k') (\mu \mathbf{s}(k') + \mathbf{\Phi}(k')) + \mathbf{w}_u(k')) e^{j\vartheta[n]} e^{j2\pi(k'-k)n/N} \\
&= \sum_{k'=0}^{N-1} (\mathbf{H}_u(k') (\mu \mathbf{s}(k') + \mathbf{\Phi}(k')) + \mathbf{w}_u(k')) \zeta(k' - k) \\
&= \mu \mathbf{H}_u(k) \mathbf{s}(k) \zeta(0) + \mu \sum_{\substack{k'=0 \\ k' \neq k}}^{N-1} \mathbf{H}_u(k') \mathbf{s}(k') \zeta(k' - k) + \sum_{k'=0}^{N-1} (\mathbf{H}_u(k') \mathbf{\Phi}(k') + \mathbf{w}_u(k')) \zeta(k' - k),
\end{aligned} \tag{5.19}$$

where the elements of $\mathbf{\Phi}(k)$ can be obtained by taking the DFT of the elements of $\overline{\mathbf{\Phi}}[n]$ and $\zeta(k)$ is given as:

$$\zeta(k) = \frac{1}{N} \sum_{n=0}^{N-1} e^{j\vartheta[n]} e^{j2\pi kn/N}. \tag{5.20}$$

Here, $\zeta(0)$ acts as CPE and $\zeta(k' - k)$ disrupts the orthogonality of the subcarriers and causes ICI. On substituting $\mathbf{\Phi}(k') = \mathbf{V}_u(k') \tilde{\mathbf{\Phi}}(k')$, where $\tilde{\mathbf{\Phi}}(k')$ is a $Q_u \times 1$ precoded nonlinear distortion vector whose elements are $\mathcal{CN}(0, N\sigma_\phi^2)$ (refer Appendix D), (5.19) can be rewritten as:

$$\begin{aligned}
\mathbf{r}_u(k) &= \mu \mathbf{H}_u(k) \mathbf{s}(k) \zeta(0) + \mu \sum_{\substack{k'=0 \\ k' \neq k}}^{N-1} \mathbf{H}_u(k') \mathbf{s}(k') \zeta(k' - k) \\
&\quad + \sum_{k'=0}^{N-1} (\mathbf{H}_u(k') \mathbf{V}_u(k') \tilde{\mathbf{\Phi}}(k') + \mathbf{w}_u(k')) \zeta(k' - k).
\end{aligned} \tag{5.21}$$

Similar to (4.37), the received decoded complex data vector can be written as:

$$\begin{aligned}
\hat{\mathbf{r}}_u(k) &= \mu \mathbf{\Xi}_u(k) \mathbf{a}_u(k) \zeta(0) + \mu \mathbf{D}_u(k)^H \sum_{\substack{k'=0 \\ k' \neq k}}^{N-1} \mathbf{D}_u(k') \mathbf{\Xi}_u(k') \mathbf{a}_u(k') \zeta(k' - k) \\
&\quad + \mathbf{D}_u(k)^H \sum_{k'=0}^{N-1} (\mathbf{D}_u(k') \mathbf{\Xi}_u(k') \tilde{\mathbf{\Phi}}(k') + \mathbf{w}_u(k')) \zeta(k' - k),
\end{aligned} \tag{5.22}$$

where $\mathbf{D}_u(k)$ is a $Q_u \times N_u$ decoding matrix at k^{th} subcarrier of u^{th} user. Since both $\mathbf{D}_u(k)$ and $\mathbf{D}_u(k')$ are unitary matrices with same dimension and using the property of orthonormal vectors, the product $\hat{\mathbf{D}}_u(k, k') = \mathbf{D}_u(k)^H \mathbf{D}_u(k')$ is also an unitary matrix. Thus, (5.22) can be rewritten

as:

$$\begin{aligned}
 \hat{\mathbf{r}}_u(k) = & \underbrace{\mu \mathbf{\Xi}_u(k) \mathbf{a}_u(k) \zeta(0)}_{\text{Desired Signal with CPE}} + \underbrace{\mu \sum_{\substack{k'=0 \\ k' \neq k}}^{N-1} \hat{\mathbf{D}}_u(k, k') \mathbf{\Xi}_u(k') \mathbf{a}_u(k') \zeta(k' - k)}_{\text{ICI}} \\
 & + \underbrace{\sum_{k'=0}^{N-1} \hat{\mathbf{D}}_u(k, k') \mathbf{\Xi}_u(k') \tilde{\mathbf{\Phi}}(k') \zeta(k' - k)}_{\text{Nonlinear Distortion}} + \underbrace{\sum_{k'=0}^{N-1} \tilde{\mathbf{w}}_u(k') \zeta(k' - k)}_{\text{AWGN}}. \quad (5.23)
 \end{aligned}$$

Remark 1 Unlike the linear system, the nonlinear MU-MIMO-OFDM system with phase noise suffers from CPE, ICI and additive nonlinear distortion which can be observed from (5.23).

Remark 2 Since the phase noise characteristic is independent of the linear transmit signal, it does not depend on the number of users which can be observed from (5.20). Thus, the number of users does not affect the performance of MU-MIMO-OFDM system impaired with phase noise.

Remark 3 The transmit precoding matrix $\mathbf{V}_u(k)$ eliminates only MUI. It does not mitigate the nonlinear distortion and phase noise.

Remark 4 Since the nonlinear distortion depends on the input signal power (P_{in}) to the HPA and P_{in} depends on the power of data signal of all users as shown in (5.2), thus on increasing the number of users, the input power P_{in} to the HPA also increases which in turn increases the nonlinear distortion. Therefore, the performance of the MU-MIMO-OFDM system with nonlinear HPA will significantly deteriorate with an increase in the number of users.

5.4 System Performance Analysis

In this section, we will evaluate the performance of the MU-MIMO-OFDM system in the presence of nonlinear distortion and phase noise in terms of SDINR and then utilize the SDINR expression to derive the closed-form expressions of SER and total capacity. The SDINR at z^{th} data stream of the MU-MIMO-OFDM system for u^{th} user can be expressed as:

$$SDINR_u^{(z)} = \frac{P_{CPE}^{(z)}}{P_{ICI}^{(z)} + P_{ND}^{(z)} + P_{AWGN}^{(z)}}, \quad (5.24)$$

where $P_{CPE}^{(z)}$ is the average power of the desired signal with CPE and attenuation factor. $P_{ICI}^{(z)}$, $P_{ND}^{(z)}$ and $P_{AWGN}^{(z)}$ are the variance of ICI, nonlinear distortion noise and AWGN, respectively. For

ease of analysis, we assume $L = 1$ which implies $\mathbf{H}_u(k) = \mathbf{H}_u \Rightarrow \mathbf{\Xi}_u(k) = \mathbf{\Xi}_u$ and $\mathbf{D}_u(k) = \mathbf{U}_u$. Hence, the average power of the desired signal with CPE $P_{CPE}^{(z)}$ and attenuation factor can be given as:

$$P_{CPE}^{(z)} = \text{var} \left[\mu \sqrt{\lambda_u^{(z)}(k)} s_u^{(z)}(k) \zeta(0) \right] = \mu^2 \lambda_u^{(z)}(k) \text{var} \left[s_u^{(z)}(k) \zeta(0) \right]. \quad (5.25)$$

The variance of product of two independent random variables A and B [123] can be given as $\text{var}[AB] = \text{var}[A]\text{var}[B] + |\mathbb{E}[A]|^2 \text{var}[B] + \text{var}[A] |\mathbb{E}[B]|^2$. Using these facts, the average power $P_{CPE}^{(z)}$ can be expanded as:

$$P_{CPE}^{(z)} = \mu^2 \lambda_u^{(z)}(k) P_u \left(\text{var}[\zeta(0)] + |\mathbb{E}[\zeta(0)]|^2 \right) = \mu^2 \lambda_u^{(z)}(k) P_u \left(\frac{1}{N} \left(1 - e^{-\sigma_\theta^2} \right) + e^{-\sigma_\theta^2} \right), \quad (5.26)$$

where the mean and variance of $\zeta(0)$ are derived in Appendix E. The variance of ICI $P_{ICI}^{(z)}$ can be expressed as:

$$\begin{aligned} P_{ICI}^{(z)} &= \mu^2 \lambda_u^{(z)}(k) \text{var} \left[\sum_{\substack{k'=0 \\ k' \neq k}}^{N-1} s_u^{(z)}(k') \zeta(k' - k) \right] \\ &= \mu^2 \lambda_u^{(z)}(k) \left(\text{var} \left[\sum_{k'=0}^{N-1} s_u^{(z)}(k') \zeta(k' - k) \right] - \text{var} \left[s_u^{(z)}(k) \zeta(0) \right] \right). \end{aligned} \quad (5.27)$$

Here,

$$\text{var} \left[\sum_{k'=0}^{N-1} s_u^{(z)}(k') \zeta(k' - k) \right] = \sum_{k'=0}^{N-1} \text{var} \left[s_u^{(z)}(k') \zeta(k' - k) \right] + I,$$

where

$$\begin{aligned} I &= \sum_{k'' \neq k'} \text{Cov} \left(s_u^{(z)}(k') \zeta(k' - k), s_u^{(z)}(k'') \zeta(k'' - k) \right) \\ &= \mathbb{E} \left[s_u^{(z)}(k') \zeta(k' - k) s_u^{(z)*}(k'') \zeta^*(k'' - k) \right] - \mathbb{E} \left[s_u^{(z)}(k') \zeta(k' - k) \right] \mathbb{E} \left[s_u^{(z)*}(k'') \zeta^*(k'' - k) \right] \\ &= \mathbb{E} \left[s_u^{(z)}(k') \right] \mathbb{E} \left[s_u^{(z)*}(k'') \right] \mathbb{E} \left[\zeta(k' - k) \zeta^*(k'' - k) \right] - \mathbb{E} \left[s_u^{(z)}(k') \right] \mathbb{E} \left[s_u^{(z)*}(k'') \right] \\ &\quad \times \mathbb{E} \left[\zeta(k' - k) \right] \mathbb{E} \left[\zeta^*(k'' - k) \right] \\ &= 0. \end{aligned} \quad (5.28)$$

Hence, (5.27) can be written as:

$$P_{ICI}^{(z)} = \mu^2 \lambda_u^{(z)}(k) \left(\sum_{k'=0}^{N-1} \text{var} \left[s_u^{(z)}(k') \zeta(k' - k) \right] - \text{var} \left[s_u^{(z)}(k) \zeta(0) \right] \right). \quad (5.29)$$

Similar to (5.26), the $P_{ICI}^{(z)}$ in (5.29) can be expressed as:

$$\begin{aligned} P_{ICI}^{(z)} &= \mu^2 \lambda_u^{(z)}(k) P_u \left(\sum_{k'=0}^{N-1} \left(\text{var} [\zeta(k' - k)] + |\mathbb{E} [\zeta(k' - k)]|^2 \right) - \left(\frac{1}{N} (1 - e^{-\sigma_\vartheta^2}) + e^{-\sigma_\vartheta^2} \right) \right) \\ &= \mu^2 \lambda_u^{(z)}(k) P_u \left(\sum_{k'=0}^{N-1} \frac{1}{N} (1 - e^{-\sigma_\vartheta^2}) + e^{-\sigma_\vartheta^2} - \left(\frac{1}{N} (1 - e^{-\sigma_\vartheta^2}) + e^{-\sigma_\vartheta^2} \right) \right) \\ &= \mu^2 \lambda_u^{(z)}(k) P_u \left(1 - \frac{1}{N} \right) (1 - e^{-\sigma_\vartheta^2}), \end{aligned} \quad (5.30)$$

where the variance of $\zeta(k' - k)$ is evaluated in Appendix F.

Lemma 1 *The variance of nonlinear distortion is independent of the phase noise and can be given as $P_{ND}^{(z)} = \lambda_u^{(z)}(k) \sigma_\phi^2$.*

Proof: The variance of nonlinear distortion $P_{ND}^{(z)}$ can be given as:

$$\begin{aligned} P_{ND}^{(z)} &= \lambda_u^{(z)}(k) \sum_{k'=0}^{N-1} \text{var} \left[\tilde{\phi}^{(z)}(k') \zeta(k' - k) \right] \\ &= \lambda_u^{(z)}(k) \sigma_\phi^2 \sum_{k'=0}^{N-1} \left(\text{var} [\zeta(k' - k)] + |\mathbb{E} [\zeta(k' - k)]|^2 \right) \\ &= \lambda_u^{(z)}(k) \sigma_\phi^2 \left(\sum_{k'=0}^{N-1} \frac{1}{N} (1 - e^{-\sigma_\vartheta^2}) + e^{-\sigma_\vartheta^2} \right) = \lambda_u^{(z)}(k) \sigma_\phi^2. \end{aligned} \quad (5.31)$$

Here, phase noise acts as phase rotation factor on the nonlinear distortion.

Lemma 2 *The variance of AWGN is independent of the phase noise and is given as:*

$$P_{AWGN}^{(z)} = \sum_{k'=0}^{N-1} \text{var} \left[\tilde{w}_u^{(z)}(k') \zeta(k' - k) \right] = N \sigma_o^2. \quad (5.32)$$

Proof: Similar to the proof of Lemma 1.

On substituting (5.26), (5.30), (5.31) and (5.32) in (5.24), the SDINR of the nonlinear MU-MIMO-OFDM system with phase noise can be written as:

$$SDINR_u^{(z)} = \frac{\mu^2 \lambda_u^{(z)}(k) P_u \left(\frac{1}{N} \left(1 - e^{-\sigma_\vartheta^2} \right) + e^{-\sigma_\vartheta^2} \right)}{\mu^2 \lambda_u^{(z)}(k) P_u \left(1 - \frac{1}{N} \right) \left(1 - e^{-\sigma_\vartheta^2} \right) + \lambda_u^{(z)}(k) \sigma_\phi^2 + N_o}. \quad (5.33)$$

Lemma 3 *The expression of SDINR is valid for any nonlinear MU-MIMO-OFDM system irrespective of the fact whether the phase noise is present at the transmitter or at the receiver or both.*

Proof: Refer to Appendix G.

Utilizing the expression of SDINR in (5.33), we will derive the closed-form expression of SER and capacity in the following subsections. Please note that for the case of MU-MIMO-OFDM system with ideal oscillators, i.e., $\sigma_\vartheta^2 = 0$, the expression of SDINR in (5.33) becomes similar to the expression obtained in [120] and to the SINR expression in (4.55) of Chapter-4 which indicates that the proposed work can also be used in evaluating the performance of MU-MIMO-OFDM system with and without nonlinear HPAs.

5.4.1 SER Performance

For each u^{th} user, the SER of the received M -QAM modulated data symbol can be given as [104]:

$$SER_u^{(z)} \approx 2 \left(1 - \frac{1}{\sqrt{M}} \right) \text{erfc} \left(\sqrt{C \frac{\lambda_u^{(z)}(k) C_u}{\lambda_u^{(z)}(k) C'_u + 1}} \right), \quad (5.34)$$

where $C = \frac{3}{2(M-1)}$, $C_u = \frac{\mu^2 P_u}{N_o} \left(\frac{1}{N} (1 - e^{-\sigma_\vartheta^2}) + e^{-\sigma_\vartheta^2} \right)$ and $C'_u = \frac{\mu^2 P_u}{N_o} \left(1 - \frac{1}{N} \right) (1 - e^{-\sigma_\vartheta^2}) + \frac{\sigma_\phi^2}{N_o}$. The averaged SER over all Q_u data streams can be written as:

$$SER_u \approx 2 \left(1 - \frac{1}{\sqrt{M}} \right) \mathbb{E}_{\lambda_u(k)} \left[\text{erfc} \left(\sqrt{C \frac{\lambda_u(k) C_u}{\lambda_u(k) C'_u + 1}} \right) \right], \quad (5.35)$$

where $E_{\lambda_u(k)}[\cdot]$ is the expectation operator conditioned on $\lambda_u(k)$ that can be randomly selected from $\lambda_u^{(z)}(k)$ whose PDF is given as [120]:

$$f_{\lambda_u(k)}(\lambda_u(k)) = \frac{1}{Q_u} \sum_{z=0}^{Q_u-1} \sum_{\gamma=0}^z \sum_{\xi=0}^{2\gamma} \frac{(-1)^\xi (2\gamma)!}{2^{2z-\xi} \gamma! \xi! (\Delta + \gamma)!} \binom{2z-2\gamma}{z-\gamma} \binom{2\gamma+2\Delta}{2\gamma-\xi} \lambda_u^{\Delta+\xi}(k) e^{-\lambda_u(k)}. \quad (5.36)$$

Thus, the SER in (5.35) can be further decomposed as:

$$SER_u \approx 2 \left(1 - \frac{1}{\sqrt{M}}\right) \frac{1}{Q_u} \sum_{z=0}^{Q_u-1} \sum_{\gamma=0}^z \sum_{\xi=0}^{2\gamma} \frac{(-1)^\xi (2\gamma)!}{2^{2z-\xi} \gamma! \xi! (\Delta + \gamma)!} \binom{2z-2\gamma}{z-\gamma} \binom{2\gamma+2\Delta}{2\gamma-\xi} I'_{u,\xi}, \quad (5.37)$$

where $\Delta = \min(A_u, N_u)$ and

$$I'_{u,\xi} = \int_0^\infty \operatorname{erfc} \left(\sqrt{C \frac{\lambda_u(k) C_u}{\lambda_u(k) C'_u + 1}} \right) \lambda_u^{\Delta+\xi}(k) e^{-\lambda_u(k)} d\lambda_u(k). \quad (5.38)$$

Using the series expansion of the $\operatorname{erfc}(\cdot)$ [105, 3.321.1], i.e., $\operatorname{erfc}(y) = 1 - \frac{2}{\sqrt{\pi}} \sum_{e=0}^\infty \frac{(-1)^e y^{2e+1}}{e!(2e+1)}$, (5.38) can be expanded as:

$$I'_{u,\xi} = 1 - \frac{2}{\sqrt{\pi}} \sum_{e=0}^\infty \frac{(-1)^e (CC_u)^{e+1/2}}{e!(2e+1)} \int_0^\infty \frac{\lambda_u^{(\Delta+\xi+e+1/2)}(k) e^{-\lambda_u(k)}}{(\lambda_u(k) C'_u + 1)^{e+1/2}} d\lambda_u(k). \quad (5.39)$$

On substituting $\psi = \lambda_u(k) C'_u$ in (5.39), we obtain

$$\begin{aligned} I'_{u,\xi} &= 1 - \frac{2}{\sqrt{\pi}} \sum_{e=0}^\infty \frac{(-1)^e (CC_u)^{e+1/2}}{e!(2e+1) C_u^{(\Delta+\xi+e+3/2)}} \int_0^\infty \frac{\psi^{(\Delta+\xi+e+1/2)} e^{-\frac{\psi}{C'_u}}}{(\psi+1)^{e+1/2}} d\psi \\ &= 1 - \frac{2}{\sqrt{\pi}} \sum_{e=0}^\infty \frac{(-1)^e (CC_u)^{e+1/2} \Gamma(\Delta+\xi+e+3/2)}{e!(2e+1) C_u^{(\Delta+\xi+e+3/2)}} U \left(\Delta+\xi+e+3/2; \Delta+\xi+2; \frac{1}{C'_u} \right), \end{aligned} \quad (5.40)$$

where $U(\cdot)$ is confluent hypergeometric function [105, 9.211.4]. Using (5.37) and (5.40), the closed-form expression³ of averaged SER for u^{th} user can be expressed as:

$$SER_u \approx \frac{2}{Q_u} \left(1 - \frac{1}{\sqrt{M}}\right) \sum_{z=0}^{Q_u-1} \sum_{\gamma=0}^z \sum_{\xi=0}^{2\gamma} \frac{(-1)^\xi (2\gamma)!}{2^{2z-\xi} \gamma! \xi! (\Delta + \gamma)!} \binom{2z-2\gamma}{z-\gamma} \binom{2\gamma+2\Delta}{2\gamma-\xi} \\ \times \left(1 - \frac{2}{\sqrt{\pi}} \sum_{e=0}^{\infty} \frac{(-1)^e (CC_u)^{e+1/2} \Gamma(\Delta + \xi + e + 3/2)}{e! (2e+1) C_u'^{(\Delta + \xi + e + 3/2)}} U\left(\Delta + \xi + e + 3/2; \Delta + \xi + 2; \frac{1}{C_u'}\right)\right). \quad (5.41)$$

Please note that the values of C_u and C_u' in (5.41) will change for MU-MIMO-OFDM system depending on the presence and absence of nonlinear HPA and phase noise as given in Table 5.1. Similar to (5.41), the closed-form expression for SER of linear MU-MIMO-OFDM system can be derived by substituting the $\mu = 1$, $\sigma_\phi^2 = 0$ and $\sigma_\theta^2 = 0$ in (5.33). Using these values, (5.38) can be rewritten as:

$$I'_{u,\xi} = \int_0^\infty \text{erfc}\left(\sqrt{\lambda_u(k) CC_u}\right) \lambda_u^{\Delta+\xi}(k) e^{-\lambda_u(k)} d\lambda_u(k). \quad (5.42)$$

Using the identity $\text{erfc}(x) = 2Q(\sqrt{2}x)$ in (5.42), we obtain

$$I'_{u,\xi} = 2 \int_0^\infty Q\left(\sqrt{2\lambda_u(k) CC_u}\right) \lambda_u^{\Delta+\xi}(k) e^{-\lambda_u(k)} d\lambda_u(k). \quad (5.43)$$

Similar to (4.47) and using (5.37) and (5.43), the closed-form SER expression of linear MU-MIMO-OFDM system can be expressed as:

$$SER_u \approx \left(1 - \frac{1}{\sqrt{M}}\right) \frac{1}{Q_u} \sum_{z=0}^{Q_u-1} \sum_{\gamma=0}^z \sum_{\xi=0}^{2\gamma} \frac{(-1)^\xi (2\gamma)! (\Delta + \xi)!}{2^{2z-\xi-1} \gamma! \xi! (\Delta + \gamma)!} \binom{2z-2\gamma}{z-\gamma} \\ \times \binom{2\gamma+2\Delta}{2\gamma-\xi} \left[1 - \sum_{d=0}^{\Delta+\xi} \binom{2d}{d} \frac{\sqrt{2CC_u}}{2^d (2CC_u + 2)^{d+1/2}}\right]. \quad (5.44)$$

³The expression in (5.41) contains the infinite series. In order to evaluate this expression, the infinite series can be truncated to $e = 10$ terms with negligible loss in precision.

Table 5.1: Value of C_u and C'_u for different MU-MIMO-OFDM systems

	In presence of Phase Noise and Nonlinear HPA	Phase Noise Only	Nonlinear HPA Only
C_u	$\frac{\mu^2 P_u}{N_o} \left(\frac{1}{N} \left(1 - e^{-\sigma_\vartheta^2} \right) + e^{-\sigma_\vartheta^2} \right)$	$\frac{P_u}{N_o} \left(\frac{1}{N} \left(1 - e^{-\sigma_\vartheta^2} \right) + e^{-\sigma_\vartheta^2} \right)$	$\frac{\mu^2 P_u}{N_o}$
C'_u	$\frac{\mu^2 P_u}{N_o} \left(1 - \frac{1}{N} \right) \left(1 - e^{-\sigma_\vartheta^2} \right) + \frac{\sigma_\vartheta^2}{N_o}$	$\frac{P_u}{N_o} \left(1 - \frac{1}{N} \right) \left(1 - e^{-\sigma_\vartheta^2} \right)$	$\frac{\sigma_\vartheta^2}{N_o}$

5.4.2 Capacity Performance

The overall average capacity of the MU-MIMO-OFDM system with nonlinear HPA and phase noise can be evaluated as:

$$Cap = \sum_{u=1}^U \mathbb{E}_{\lambda_u(k)} \left[\log_2 \left(1 + \frac{\lambda_u(k) C_u}{\lambda_u(k) C'_u + 1} \right) \right]. \quad (5.45)$$

Utilizing (5.36), (5.45) can be rewritten as:

$$Cap = \frac{1}{Q_u} \sum_{u=1}^U \sum_{z=0}^{Q_u-1} \sum_{\gamma=0}^z \sum_{\xi=0}^{2\gamma} \frac{(-1)^\xi (2\gamma)!}{2^{2z-\xi} \gamma! \xi! (\Delta + \gamma)!} \binom{2z-2\gamma}{z-\gamma} \binom{2\gamma+2\Delta}{2\gamma-\xi} \\ \times \int_0^\infty \log_2 \left(1 + \frac{\lambda_u(k) C_u}{\lambda_u(k) C'_u + 1} \right) \lambda_u^{\Delta+\xi}(k) e^{-\lambda_u(k)} d\lambda_u(k). \quad (5.46)$$

Exploiting the identity $\ln(ab) = \ln(a) + \ln(b)$ and [105, 4.337.5], the closed-form expression of overall capacity can be given as:

$$Cap = \frac{1}{Q_u \ln(2)} \sum_{u=1}^U \sum_{z=0}^{Q_u-1} \sum_{\gamma=0}^z \sum_{\xi=0}^{2\gamma} \frac{(2\gamma)!}{2^{2z-\xi} \gamma! \xi! (\Delta + \gamma)!} \binom{2z-2\gamma}{z-\gamma} \binom{2\gamma+2\Delta}{2\gamma-\xi} \sum_{b=0}^{\Delta+\xi} (-1)^{\Delta-b-1} \\ \times \frac{(\Delta + \xi)!}{(\Delta + \xi - b)!} \left(\frac{\text{Ei} \left(\frac{-1}{C_u + C'_u} \right) e^{\frac{1}{C_u + C'_u}}}{(C_u + C'_u)^{\Delta+\xi-b}} - \frac{\text{Ei} \left(\frac{-1}{C'_u} \right) e^{\frac{1}{C'_u}}}{(C'_u)^{\Delta+\xi-b}} + \sum_{f=1}^{\Delta+\xi-b} \frac{(f-1)!}{(-1)^{f-1}} \right. \\ \left. \left(\left(\frac{1}{C_u + C'_u} \right)^{\Delta+\xi-b-f} - \left(\frac{1}{C'_u} \right)^{\Delta+\xi-b-f} \right) \right), \quad (5.47)$$

The closed-form expression of capacity (5.47) can be used to evaluate the performance of MU-MIMO-OFDM system in the presence and absence of phase noise and nonlinear HPAs by substituting the corresponding values of C_u and C'_u that are listed in Table 5.1. However, the closed-form equation of overall capacity of linear MU-MIMO-OFDM system can be expressed

Table 5.2: Value of the different parameters

Parameter	N	N_g	A_{sat}	α	α_θ	IBO
Value	1024	256	P_T/N_T	1	0	5 dB

as:

$$\begin{aligned}
Cap = & \frac{1}{Q_u \ln(2)} \sum_{u=1}^U \sum_{z=0}^{Q_u-1} \sum_{\gamma=0}^z \sum_{\xi=0}^{2\gamma} \frac{(2\gamma)!}{2^{2z-\xi} \gamma! \xi! (\Delta + \gamma)!} \binom{2z-2\gamma}{z-\gamma} \binom{2\gamma+2\Delta}{2\gamma-\xi} \sum_{b=0}^{\Delta+\xi} (-1)^{\Delta-b-1} \\
& \times \frac{(\Delta + \xi)!}{(\Delta + \xi - b)!} \left(\frac{\text{Ei}\left(-\frac{1}{C_u}\right) e^{\frac{1}{C_u}}}{(C_u)^{\Delta+\xi-b}} + \sum_{f=1}^{\Delta+\xi-b} \frac{(f-1)!}{(-1)^{f-1}} \left(\frac{1}{C_u}\right)^{\Delta+\xi-b-f} \right). \quad (5.48)
\end{aligned}$$

The closed-form expressions of SER and capacity in (5.41) and (5.47), respectively, provide the complete performance analysis of MU-MIMO-OFDM system in the presence of hardware impairments such as phase noise and nonlinear HPA⁴. It can also be noted for the case of nonlinear HPA only, all expressions obtained in this chapter are similar to expressions obtained in chapter-4. Thus, we can say the framework analyzed in chapter-4 is the special case of this chapter.

5.5 Simulation Results and Discussion

In this section, we present the simulation results to validate the proposed analytical framework for the MU-MIMO-OFDM system with nonlinear HPA and phase noise. In the first subsection, we analyze the impact of phase noise on the SER performance of the MU-MIMO-OFDM system with and without nonlinear HPAs. In the subsequent subsection, we present the joint impact of phase noise and HPA nonlinearity on the overall capacity of the system. Throughout this section, we represent the MU-MIMO system via $(N_T, [N_1, N_2, \dots, N_U])$ and the SU-MIMO system via (N_T, N_R) . For ease of simulation, we assume that all the users have the same number of receiving antennas, i.e., $N_u = N_R \forall u \in [1, U]$ and receive the data via same number of parallel data streams, i.e., $Q_u = Q \forall u \in [1, U]$. For a fair comparison, the total power constraint in SU and MU systems is applied, and total power is kept to P_T . Other common parameters that are used in this section are listed in Table 5.2. As a performance benchmark, we also show the results of the linear system which consists of linear HPAs and ideal oscillators (no phase noise). Since the performance of the MU-MIMO-OFDM system with nonlinear HPAs are extensively

⁴As mentioned Chapter-3, the closed-form expression for SER is derived for high SNR values but not applicable for low SNR values. The SER is calculated numerically using (3.24) for low SNR values.

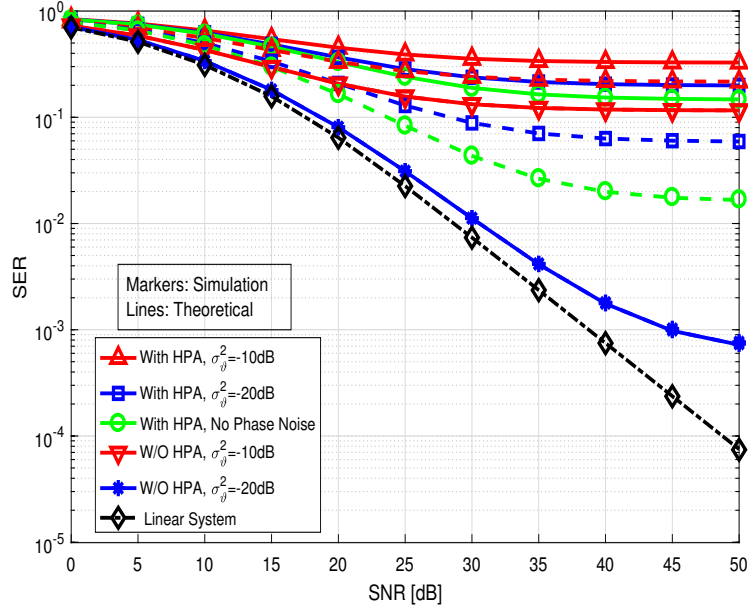


Figure 5.2: An average SER performance comparison between (6,[2,2,2]) MU-MIMO-OFDM system (solid lines) and (2,2) SU-MIMO-OFDM system (dashed lines) for different phase noise.

stimulated in the literature [28, 120, 124], thus in this section, we primarily focus on the impact of phase noise on the MU-MIMO-OFDM system in the absence and presence of nonlinear HPAs.

5.5.1 Error Performance

The average SER performance of (6,[2,2,2]) MU-MIMO-OFDM and (2,2) SU-MIMO-OFDM systems are compared for different AWGN SNR (P_T/N_o) in Fig. 5.2. Here, each user data is transmitted over two parallel streams, i.e., $Q = 2$. It is evident from Fig. 5.2 that the theoretical and simulation results are in good agreement with each other which verifies the analytically obtained SER expression. It can be observed that HPA nonlinearity and phase noise significantly deteriorate the SER performance of both systems. One of the reasons for performance degradation is that the ICI caused by phase noise which leads to an irreducible error floor. However, this error floor depends on the statistics of the phase noise. For instance, SER performance get saturated around 10^{-1} for $\sigma_\phi^2 = -10$ dB, whereas its saturation level reduces to 10^{-3} for $\sigma_\phi^2 = -20$ dB. In the absence of nonlinear HPA, the error performance of the both MU and SU systems is similar for same phase noise variance which can be observed from Fig. 5.2⁵. This is due to the fact that the phase noise characteristics are independent

⁵The error performance curves of MU and SU systems in the presence of same phase noise and ideal HPA are overlapped on each other. Thus, it is difficult to visualize them. To get the more clarification, please refer Fig. 5.3 and Fig. 5.4.

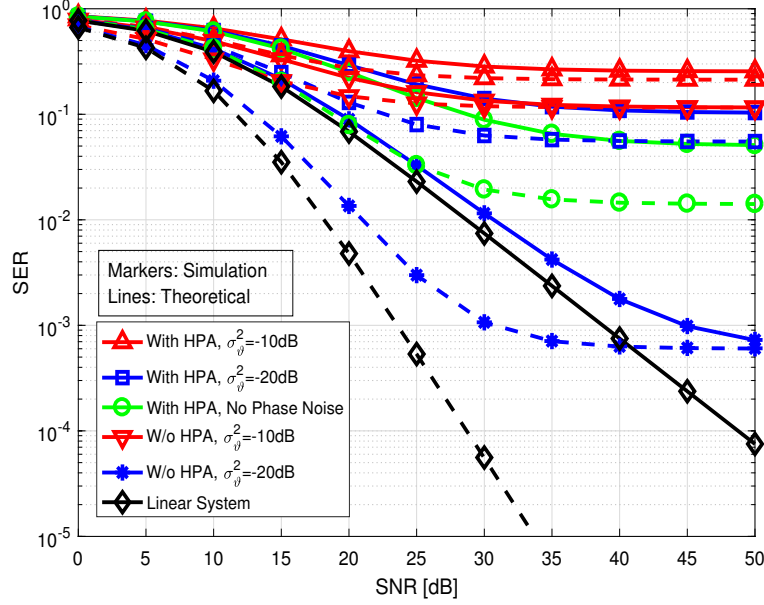


Figure 5.3: An average SER performance comparison between $(3,[1,1,1])$ system (solid lines) and $(3,[1,1])$ system (dashed lines) for different phase noise with and without HPA nonlinearity.

of the linear transmit signal, thus it does not depend on the number of users which is also discussed in the *Remark 2*. Therefore, the number of users does not affect the SER performance of MU-MIMO-OFDM system impaired with phase noise only. It is worth noticing that the error performance of the MU system is worse than the SU system in the presence of nonlinear HPA. This is due to the fact that the nonlinear distortion noise depends on the number of users which is also discussed in the *Remark 4*. It may be noted that in the absence of phase noise, the SER performance of both systems with nonlinear HPAs is similar to that obtained in [120].

The joint impact of HPA nonlinearity and phase noise on the MU system for the different number of users is shown in Fig. 5.3. Two MU systems, $(3,[1,1,1])$ and $(3,[1,1])$ are considered with single data stream. Due to the different transmit diversity in both the systems, there is a considerable gap between the error performance of both systems impaired with phase noise in the low SNR regime. However, in high SNR regime, ICI dominates the AWGN ($P_{ICI} \gg P_{AWGN}$) in both systems which leads to same irreducible error floor as shown in Fig. 5.3. Since the only source of ICI is the presence of phase noise, this strengthens the fact that the impact of phase noise is independent of the number of users. It can also be observed in Fig. 5.3 that the error floor of both systems gets saturated at different levels in the presence of both phase noise and nonlinear HPAs. This is due to the dependency of the nonlinear distortion on the number of users.

Fig. 5.4 shows the impact of the variance of phase noise and HPA nonlinearity on the error performance of the MU system with the different number of users. Here, we have fixed

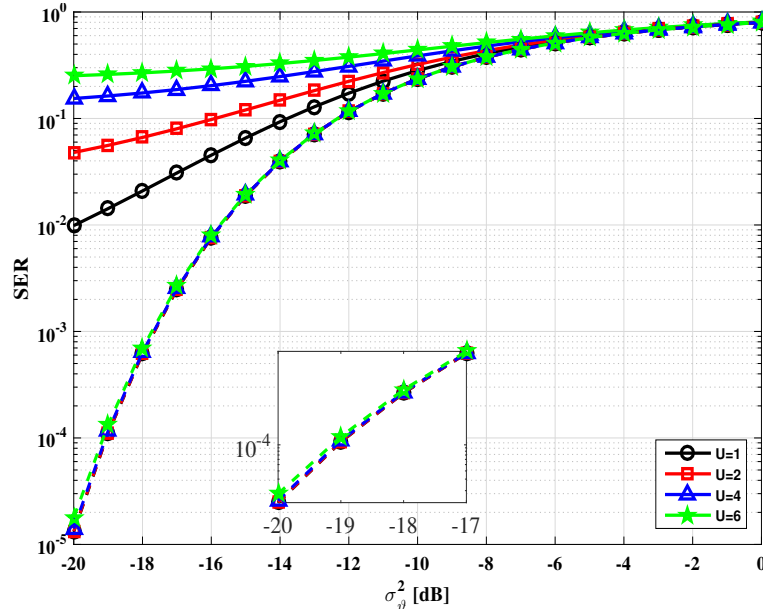


Figure 5.4: Solid lines and dashed lines represent an average SER performance of the MU-MIMO-OFDM system with and without nonlinear HPA, respectively, versus the variance of phase noise.

SNR=30dB, $N_T = 15$, $N_R = 2$, and $Q = 2$. It can easily be inferred from Fig. 5.4 that the SER performance is same for any number of users accommodated by the system in the absence of nonlinear HPA. However, the error performance deteriorates with an increase in the number of users in the presence of nonlinear HPA and saturates at different levels.

It can be concluded from Fig. 5.2-5.4 that the impact of phase noise is independent of the number of users which implies that increase or decrease in the number of users at any instant has no bearing on the SER performance of MU-MIMO-OFDM system with linear HPAs. However, it is also evident from these results that HPA nonlinearity depends on the number of users in the system, and the error performance degrades with an increase in the number of users in the presence of nonlinear HPAs. In addition, the SER performance of the MU-MIMO-OFDM system becomes worse in the presence of both impairments. The conventional transmit precoding scheme is inadequate in mitigating the nonlinear distortion and phase noise. This is due to the fact that the precoding scheme only takes into account the wireless channel conditions thus eliminating the MUI only.

5.5.2 Capacity Performance

Throughout this section, it is assumed that BS is equipped with $N_T = 15$ transmit antennas and each user has $N_R = 2$ receive antennas with noise variance $N_o = 0$ dB. Further, equal power is allocated for each user at BS.

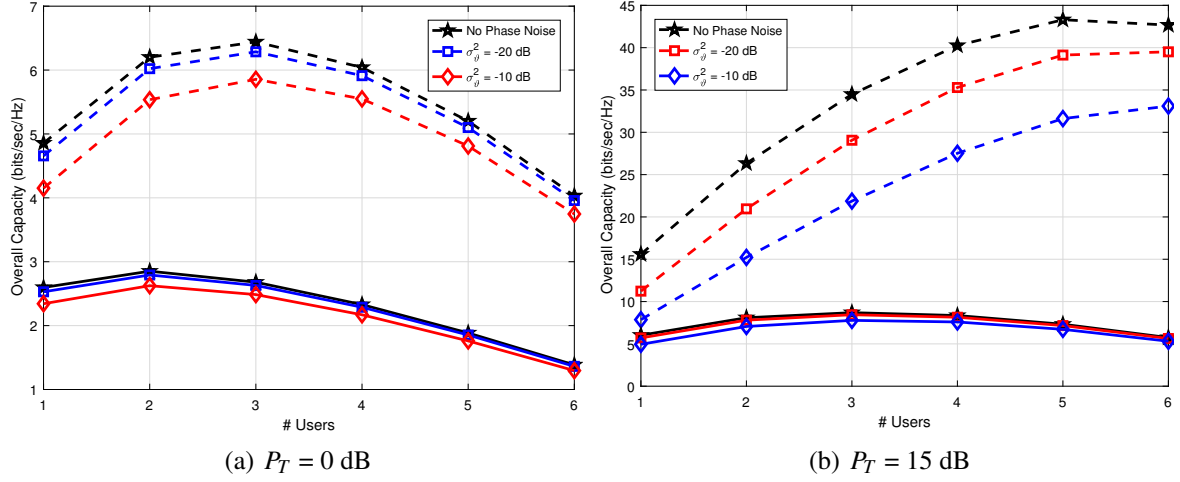


Figure 5.5: Theoretical (lines) and simulation (markers) results of overall capacity of MU-MIMO-OFDM system with (solid lines) and without (dashed lines) nonlinear HPA for different total power constraint.

Fig. 5.5 shows the capacity performance of the MU-MIMO-OFDM system in the presence of phase noise with and without nonlinear HPA for the different number of users and total transmit power P_T . It can be observed that the theoretical and simulation results are in excellent agreement which validates the proposed theoretical framework for the MU-MIMO-OFDM system. Due to the trade-off between MUI elimination and transmit diversity [120], the capacity of the MU-MIMO system does not always increase with an increase in the number of users which can easily be observed from Fig. 5.5.

In the absence of HPA nonlinearity, the capacity of MU system degrades with an increase in the phase noise variance which is also evident from Fig. 5.5. This is due to the fact that larger the phase noise, more is the ICI which reduces the SDINR of the system. However, the trade-off between the MUI elimination and transmit diversity remains same irrespective of the presence of phase noise in the system. This is contradictory to the case when there is HPA nonlinearity. In the presence of nonlinear HPA, the capacity performance of the MU system considerably degrades and further deteriorates in the presence of the phase noise. Furthermore, HPA nonlinearity also increases the aforementioned trade-off and reduces the number of users to attain maximum capacity which can easily be observed from Fig. 5.5.

The complete analysis of the capacity performance of the MU-MIMO-OFDM system for the different number of users and phase noise is shown in Fig. 5.6. The joint impact of HPA nonlinearity and phase noise drastically degrades the capacity performance of the MU system. It can be observed that on decreasing the variance of phase noise or in other words, on reducing the impact of phase noise, the total capacity starts increasing and saturates to its maximum value that can be achieved in the absence of phase noise. However, capacity reaches to its

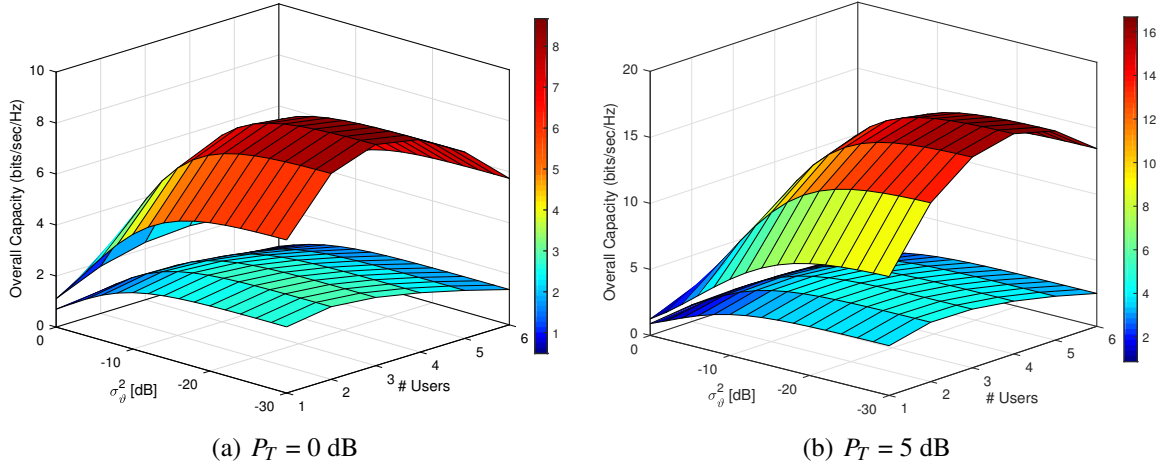


Figure 5.6: Overall capacity of MU-MIMO-OFDM system with (lower curve) and without (upper curve) nonlinear HPA with different number of users and different variance of phase noise for different total power constraint.

maximum value earlier in case of $P_T = 0$ dB as compared to the case of $P_T = 5$ dB. This is due to the fact that the ICI depends on P_u^2 which further depends on the total transmit power P_T , thus for low P_T , the ICI is also low. As a result, the impact of ICI vanishes out earlier for lower total transmit power with a decrease in the variance of phase noise. It implies that the system with low P_T can tolerate higher phase noise than the system with high P_T . A similar observation can be made in the presence of nonlinear HPA in Fig. 5.6.

From Fig. 5.5 and Fig. 5.6, it can be concluded that on increasing the total transmit power, more number of users can be accommodated in the system to attain maximum capacity. However, high transmit power also leads to more ICI which adversely affects the capacity in the presence of the phase noise. It indicates that high phase noise can be tolerated with a low total transmit power at the expense of the lower number of users to attain the maximum capacity of the MU-MIMO-OFDM system. Thus, there is a trade-off between the phase noise tolerance and the number of users that can be accommodated simultaneously to attain maximum capacity. The same trade-off can also be observed in the presence of the nonlinear HPA.

5.6 Conclusion

This work proposes an analytical methodology to study the joint impact of non-ideal oscillators and nonlinear HPAs on the performance of the MU-MIMO-OFDM system. The impact has been investigated by deriving the closed-form expressions for SER and capacity of the MU-MIMO-OFDM system in the presence of both phase noise and nonlinear distortion caused by

non-ideal oscillators and nonlinear HPAs, respectively. It has been shown that by utilizing these derived expressions, it is possible to evaluate the performance of MU-MIMO-OFDM system in the presence or absence of any of these impairments. Specifically, for the case when only phase noise is present, the error performance of the aforementioned system remains unchanged irrespective of the number of users. However, in the presence of nonlinear HPA, the error performance of the MU system significantly deteriorates with an increase in the number of users. Furthermore, it has been shown that the joint impact causes an irreducible error floor. This error floor is significantly higher than the error floor caused due to the impact of nonlinear HPA only.

Our results also show that the joint impact of phase noise and nonlinear distortion also significantly reduces the capacity of the MU-MIMO-OFDM system as compared to the ideal system. The capacity performance can be improved by operating HPA with higher back-off. However, high back-off suppresses the nonlinear distortion but conversely reduces the transmit power of each antenna, which in turn may adversely affects the capacity of the MU system. On the other hand, it has also been observed that the ICI caused by the phase noise can also be reduced by reducing the total transmit power of the system which in turn reduces the number of users that can be supported by the system to attain maximum system capacity.

To summarize, there are three trade-offs associated with the MU-MIMO-OFDM system in the presence of hardware impairments: 1) Trade-off between MUI elimination and transmit diversity, 2) Trade-off between operating HPA with high back-off and power efficiency, 3) Trade-off between phase noise tolerance and the number of users to achieve maximum capacity. These trade-offs severely restrict the performance gains of the MU-MIMO-OFDM system. As a consequence, the impact of the system hardware impairments such as non-ideal oscillators and nonlinear HPAs should be taken into consideration while evaluating the performance of the MU-MIMO-OFDM system.

Performance of Cascaded DPD+HPA Architecture

In this chapter, the impact of cascaded DPD+HPA architecture on the error performance of the MIMO-OFDM system has been studied. The brief introduction and contributions related to above mentioned system are given in Section 6.1 followed by Section 6.2 which presents the system model. Section 6.3 explains the cascade chain of the DPD and HPA mathematically. The derivation of SER is provided in Section 6.4. The numerical and simulation results are presented in Section 6.5. Finally, Section 6.6 concludes the chapter.

6.1 Introduction

HPAs are one of the vital components in the wireless communication systems. The power efficiency and nonlinear distortion are quite apparent in the HPAs. Although the HPA exhibits high power efficiency with a high input power signal, it forces the HPA to operate in a nonlinear region which inherently degrades the transmit signal quality. However, HPA can be operated in a linear region with a low power input signal which results in poor power efficiency. Nonlinear distortion causes in-band and out-of-band distortions, where in-band distortion degrades the reliability and capacity of the wireless communication system whereas out-of-band distortion results in spectral regrowth and ACPR. The most trivial solution to mitigate this nonlinear distortion is to operate the HPA with IBO. This method assures that the HPA operates in the linear region and fulfills the EVM requirements [111, 112]. However, it results in poor efficiency. Another solution to mitigate the HPA nonlinearity is the linearization techniques [54, 56, 62, 125]. Most prominent among them is DPD, since it provides high efficiency and flexible implementation at a lower cost [54]. It has inverse nonlinear characteristics as compared

to the HPA. It modifies the transmitted signal in such a manner that the resultant cascade of DPD and HPA output will be linear in nature. Consequently, it reduces the spectral regrowth and ACPR. However, the performance of the RF transmitters in the presence of DPD is limited to evaluating the spectral regrowth and ACPR which are caused by the out-of-band distortion, without much consideration on the system reliability which generally depends on the in-band distortions. In addition, most of the existing literature is restricted to theoretically analyzing the performance of the multi-carrier wideband communication systems due to nonlinear HPA only.

Motivated by the above, this chapter presents an end-to-end analytical framework to evaluate the performance of the cascaded DPD+HPA architecture on the MIMO-OFDM system. A JNP model is proposed to represent the characteristics of the cascaded DPD+HPA architecture. This model is used to analyze the information transmitted through the cascaded DPD+HPA architecture. It is shown that the received symbol after the MIMO-OFDM demodulation consists of residual noise and a multiplicative factor. The residual noise depends on the nonlinearity order of HPA and can be reduced by increasing the nonlinearity order of DPD. The multiplicative factor depends on the gain of the HPA and DPD. Hence, by modeling the residual noise as a complex Gaussian process, an analytical methodology is adopted to obtain the received SDNR of a nonlinear MIMO-OFDM system in the presence of the DPD. Further by utilizing the expression of SDNR, a closed-form expression of SER is also derived for a multi-path Rayleigh fading channel. It is shown that the SER of the MIMO-OFDM system in the presence of cascaded DPD+HPA architecture significantly improves and approaches to a linear MIMO-OFDM system. In addition, the performance of the cascaded DPD+HPA architecture is also evaluated in terms of SDR, EVM and normalized mean square error (NMSE). Simulation results are also presented to validate the efficacy of the proposed analysis.

6.2 System Model

We have considered a MIMO-OFDM system with N_T transmit antennas and N_R receive antennas as shown in Fig. 6.1. Each transmitting chain has three fundamental blocks: OFDM modulator, DPD, and HPA which will be discussed in the subsequent subsections. The MIMO channel $\bar{\mathbf{H}}[n]$ between transmitter and receiver is assumed to be a WSSUS Rayleigh fading channel [66], which can be modeled as:

$$\bar{\mathbf{H}}[n] = \sum_{l=0}^{L-1} \bar{\mathbf{H}}^l \delta[n-l], \quad (6.1)$$

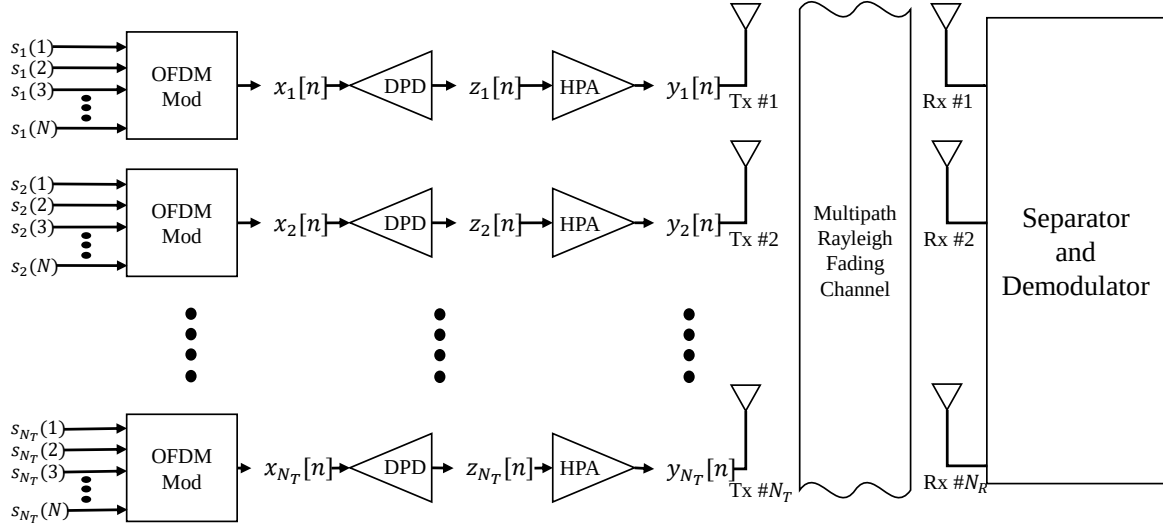


Figure 6.1: A schematic of the nonlinear MIMO-OFDM system in the presence of DPD.

where L is total number of delay taps, and $\bar{\mathbf{H}}^l$ contains the MIMO channel coefficients for the l^{th} delay path which can be written as

$$\bar{\mathbf{H}}^l = \begin{bmatrix} h_{1,1,l} & h_{1,2,l} & \cdot & \cdot & h_{1,N_T,l} \\ h_{2,1,l} & h_{2,2,l} & \cdot & \cdot & h_{2,N_T,l} \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ h_{N_R,1,l} & h_{N_R,2,l} & \cdot & \cdot & h_{N_R,N_T,l} \end{bmatrix}_{N_R \times N_T} \quad (6.2)$$

where $h_{i,t,l} \sim \mathcal{CN}(0, 1)$ are the channel impulse response between t^{th} transmitting antenna and i^{th} receive antenna for l^{th} delay tap.

6.2.1 OFDM Modulator

The OFDM modulator consists of IDFT block which converts the frequency domain data into a time domain data. After IDFT, a cyclic prefix is added to mitigate ISI followed by a parallel-to-serial converter. The corresponding baseband equivalent expression of the OFDM symbol with N subcarriers at the discrete time index n can be written as [65]:

$$x^t[n] = \begin{cases} \frac{1}{N} \sum_{k=0}^{N-1} s^t(k) e^{j2\pi kn/N}, & -N_g \leq n \leq N-1 \\ 0 & \text{otherwise} \end{cases} \quad (6.3)$$

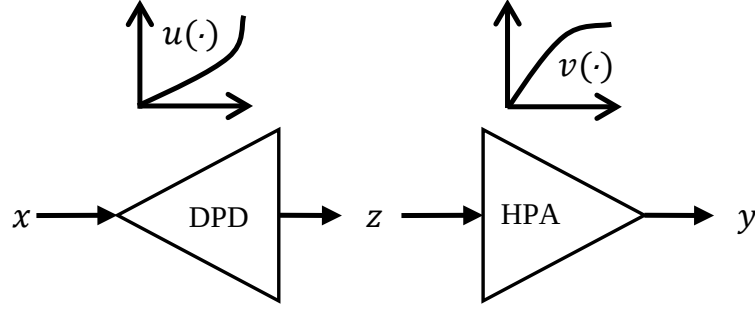


Figure 6.2: A characteristics of DPD and HPA at the transmitter.

where $x^t[n]$ is the OFDM symbol input to the DPD of t^{th} transmitting antenna, $s^t(k)$ is the complex data symbol on k^{th} subcarrier of the OFDM symbol and N_g is the cyclic prefix length. Without loss of generality, it has been assumed that complex data symbols $s^t(k)$ are i.i.d. with zero mean and variance P_s . By CLT [101], when the number of subcarriers N is large, $x^t[n]$ can be assumed to be a complex Gaussian process with zero mean and variance $P_{in} = P_s/N$.

6.2.2 Digital Predistortor

The typical characteristics of DPD and HPA are shown in Fig. 6.2. DPD is generally used to mitigate the nonlinear distortion caused by HPA. Specifically, DPD provides a complimentary characteristics as that of the HPA, prior to the input of the HPA. Thus, the output of a cascaded DPD+HPA should ideally be a scaled replica of the original signal. The characteristics of DPD are usually modeled by nonlinear polynomial model which can be expressed as [41, 66]:

$$z^t[n] = \sum_{p=1}^P \beta_p^t x^t[n] |x^t[n]|^{p-1}, \quad (6.4)$$

where $z^t[n]$ is the DPD output at t^{th} transmitting antenna when $x^t[n]$ is given as input. β_p^t are the complex nonlinearity coefficients of DPD for p^{th} order nonlinearity and P is the DPD nonlinearity order. In this work, the nonlinear coefficients of DPD are estimated using indirect learning architecture approach [56].

6.2.3 High Power Amplifier

The nonlinear characteristics of the HPA can also be modeled by nonlinear polynomial model and its baseband equivalent representation can be expressed as [41, 66]:

$$y^t[n] = \sum_{c=1}^C \alpha_{2c-1}^t z^t[n] |z^t[n]|^{2(c-1)}, \quad (6.5)$$

where $z^t[n]$ and $y^t[n]$ are the input and output of the nonlinear HPA of the t^{th} transmitting antenna, respectively. C denotes the nonlinearity order and α_{2c-1}^t are the complex nonlinearity coefficients of HPA for $(2c-1)^{\text{th}}$ order nonlinearity. Due to intermodulation products (IM3 and IM5 components), maximum in-band distortion is caused by odd order nonlinearity. Thus, odd order nonlinearities are mainly considered in characterizing the nonlinear behavior of HPA [41].

6.3 Cascaded DPD+HPA Architecture

The OFDM signal is passed through the cascaded chain of DPD and HPA. The coefficients of DPD in each transmit chain are estimated separately depending upon the nonlinearity of the corresponding HPA. Using (6.5) and (6.4), the transmit signal at the t^{th} transmit antenna can be represented by JNP model which is expressed as:

$$y^t[n] = \sum_{c=1}^C \alpha_{2c-1}^t \left(\sum_{p=1}^P \beta_p^t x^t[n] |x^t[n]|^{p-1} \right) \left| \sum_{p=1}^P \beta_p^t x^t[n] |x^t[n]|^{p-1} \right|^{2(c-1)}. \quad (6.6)$$

After rearranging the terms, (6.6) can be rewritten as:

$$y^t[n] = \sum_{g=1}^{P(2C-1)} \delta_g^t x^t[n] |x^t[n]|^{g-1}, \quad (6.7)$$

where δ_g^t are the complex nonlinear coefficients of JNP model. The total number of coefficients required to realize (6.7) is $P(2C-1)$ which depends on the nonlinearity order of the both DPD and HPA. Thus, even for small values of P and C , a large number of coefficients will be required. For instance, if $P = 7$ and $C = 4$ then total number of coefficients required to realize (6.7) is $7 \times (2 \times 4 - 1) = 49$. If $P = 6$ and $C = 3$, then 30 coefficients are required. On substituting, $P = 1$ and $\beta_1^t = 1$ in (6.6), it reduces to (6.5). This implies that the proposed theoretical framework is also valid for the nonlinear MIMO-OFDM systems without DPD. For ease of analysis, we consider the HPA with 3rd order nonlinearity ($C = 2$) and DPD with 6th

order nonlinearity ($P = 6$). Therefore, the total number of coefficients δ_g^t required is 18 and their expressions are given in Appendix H.

6.4 Received Signal Characterization

The received signal vector at the receiver of the MIMO-OFDM system can be expressed as:

$$\bar{\mathbf{r}}[n] = \sum_{l=0}^{L-1} \bar{\mathbf{H}}^l \bar{\mathbf{y}}[n-l] + \bar{\mathbf{w}}[n], \quad (6.8)$$

where $\bar{\mathbf{r}}[n]$ is a $N_R \times 1$ vector of received signals, $\bar{\mathbf{y}}[n]$ is a $N_T \times 1$ transmit signal vector at the output of cascade chain of DPD and HPA in transmitter, $\bar{\mathbf{w}}[n]$ is a $N_R \times 1$ noise vector whose components $n_o^r \sim \mathcal{CN}(0, N_o)$ are the AWGN. Using (6.7), the received complex data symbol at the i^{th} receive antenna can be written as:

$$r^i[n] = \sum_{t=0}^{N_T-1} \sum_{l=0}^{L-1} h_{i,t,l} \sum_{g=1}^{P(2C-1)} \delta_g^t x^t[n-l] |x^t[n-l]|^{g-1} + w^i[n]. \quad (6.9)$$

On removing the cyclic prefix and then applying DFT operation, we obtain

$$\begin{aligned} r^i(k) &= \sum_{n=0}^{N-1} r^i[n] e^{-j2\pi kn/N} \\ &= \sum_{n=0}^{N-1} \sum_{t=0}^{N_T-1} \sum_{l=0}^{L-1} h_{i,t,l} \sum_{g=1}^{P(2C-1)} \delta_g^t x^t[n-l] |x^t[n-l]|^{g-1} e^{-j2\pi kn/N} + w^i(k), \end{aligned} \quad (6.10)$$

where $w^i(k)$ is the AWGN in k^{th} subcarrier at i^{th} receive antenna. Using circular time shift property of DFT, the received complex data symbols at k^{th} subcarrier can be expressed as:

$$\begin{aligned} r^i(k) &= \sum_{t=0}^{N_T-1} \sum_{l=0}^{L-1} h_{i,t,l} \sum_{n=0}^{N-1} \sum_{g=1}^{P(2C-1)} \delta_g^t x^t[n] |x^t[n]|^{g-1} e^{-j2\pi k(n+l)/N} + w^i(k) \\ &= \sum_{t=0}^{N_T-1} \tilde{h}_{i,t,k} b^t(k) + w^i(k), \end{aligned} \quad (6.11)$$

where $\tilde{h}_{i,t,k}$ is the effective channel coefficient from t^{th} transmit antenna to i^{th} receive antenna of k^{th} subcarrier and can be written as:

$$\tilde{h}_{i,t,k} = \sum_{l=0}^{L-1} h_{i,t,l} e^{-j2\pi kl/N}, \quad (6.12)$$

and $b^t(k)$ is the effective nonlinear complex data symbol, i.e.,

$$b^t(k) = \sum_{n=0}^{N-1} \sum_{g=1}^{P(2C-1)} \delta_g^t x^t[n] |x^t[n]|^{g-1} e^{-j2\pi kn/N}. \quad (6.13)$$

The complex data symbol $b^t(k)$ in (6.13) can be canonically decomposed into multiplicative factor μ^t multiplied to the desired data symbol $s^t(k)$ and additive residual noise γ^t , and can be expressed as [20]:

$$b^t(k) = \mu^t s^t(k) + \gamma^t. \quad (6.14)$$

The multiplicative factor μ^t acts as gain provided by the cascade chain of DPD and HPA. On considering the μ^t as an unknown parameter, it can be expressed as [20]:

$$E[b^t(k)s^{t*}(k)] = \mu^t P_s \quad (6.15)$$

Substituting $b^t(k)$ from (6.13) and $s^{t*}(k)$ by its IDFT, we obtain

$$E[b^t(k)s^{t*}(k)] = E \left[\sum_{n=0}^{N-1} \sum_{g=1}^{P(2C-1)} \delta_g^t x^t[n] |x^t[n]|^{g-1} e^{-j2\pi kn/N} \sum_{u=0}^{N-1} x^{t*}[u] e^{j2\pi uk/N} \right]. \quad (6.16)$$

Since $x^t[n_1]$ and $x^t[n_2]$ are jointly Gaussian and uncorrelated, thus they are independent as well. Moreover, $|x^t[n]|$ is Rayleigh distributed random variable whose central moment is known [68]. Thus, (6.16) can be rewritten as:

$$E[b^t(k)s^{t*}(k)] = \begin{cases} \sum_{n=0}^{N-1} \sum_{g=1}^{P(2C-1)} \delta_g^t E[|x^t[n+g]|^{g+1}] & n = u \\ 0 & n \neq u \end{cases} \quad (6.17)$$

After some manipulations and using (6.15) and (6.17), μ^t can be expressed as:

$$\mu^t = \sum_{g=1}^{P(2C-1)} \delta_g^t \Gamma \left(1 + \frac{g+1}{2} \right) (P_{in})^{\frac{g-1}{2}}. \quad (6.18)$$

The additive residual noise γ^t can be expressed as:

$$\gamma^t = \sum_{n=0}^{N-1} \sum_{g=1}^{P(2C-1)} \delta_g^t x^t[n] |x^t[n]|^{g-1} e^{-j2\pi kn/N} - s^t(k) \sum_{g=1}^{P(2C-1)} \delta_g^t \Gamma \left(1 + \frac{g+1}{2} \right) (P_{in})^{\frac{g-1}{2}} \quad (6.19)$$

Since, the input signal $x^t[n]$ is modeled as complex Gaussian process and the expression of the residual noise in (6.19) is similar as given in [113], therefore the residual noise can be estimated

as a zero mean complex Gaussian process which is uncorrelated with the transmitted symbol $s^t(k)$ for a sufficiently large value of N . The variance $\sigma_{\gamma^t}^2$ of the residual noise can be given as:

$$\sigma_{\gamma^t}^2 = E[|\gamma^t|^2] = E[|b^t(k)|^2] - P_s |\mu^t|^2. \quad (6.20)$$

Using (6.13), we have

$$E[|b^t(k)|^2] = \sum_{n=0}^{N-1} \sum_{g=1}^{P(2C-1)} \sum_{n'=0}^{N-1} \sum_{g'=1}^{P(2C-1)} \delta_g^t \delta_{g'}^{t*} e^{-j2\pi k(n-n')/N} E[x^t[n] |x^t[n]|^{g-1} x^{t*}[n'] |x^t[n']|^{g'-1}] \quad (6.21)$$

If $n \neq n'$, $E[\cdot]$ in (6.21) can be expressed as:

$$E[x^t[n] |x^t[n]|^{g-1} x^{t*}[n'] |x^t[n']|^{g'-1}] = E[x^t[n] |x^t[n]|^{g-1}] E[x^{t*}[n'] |x^t[n']|^{g'-1}] \quad (6.22)$$

As stated earlier, $x^t[n]$ is circularly symmetric complex-valued Gaussian random variable and $|x^t[n]|^p$ is a random positive number. Therefore, the new random variable $x^t[n] |x^t[n]|^p$ is also circularly symmetric and its mean is zero [20, 68]. Thus,

$$E[x^t[n] |x^t[n]|^p] = 0 \quad (6.23)$$

Using (6.22), (6.23) and the central moment of $|x^t[n]|$, the expression in (6.21) can be expressed as:

$$E[|b^t(k)|^2] = \begin{cases} N \sum_{g=0}^{P(2C-1)} \sum_{g'=0}^{P(2C-1)} \delta_g^t \delta_{g'}^{t*} \Gamma\left(1 + \frac{g+g'}{2}\right) (P_{in})^{\frac{g+g'}{2}} & n = n' \\ 0 & n \neq n' \end{cases} \quad (6.24)$$

On substituting (6.18) and (6.24) in (6.20), the variance of the residual noise γ^t can be expressed as:

$$\sigma_{\gamma^t}^2 = N \sum_{g_1=1}^{P(2C-1)} \sum_{g_2=1}^{P(2C-1)} \delta_{g_1}^t \delta_{g_2}^{t*} (P_{in})^{\frac{g_1+g_2}{2}} \left[\Gamma\left(1 + \frac{g_1+g_2}{2}\right) - \Gamma\left(1 + \frac{g_1+1}{2}\right) \Gamma\left(1 + \frac{g_2'+1}{2}\right) \right] \quad (6.25)$$

Using (6.11) and (6.14), the received complex data symbol $r^i(k)$ can be written as:

$$r^i(k) = \sum_{t=0}^{N_T-1} \tilde{h}_{i,t,k} (\mu^t s^t(k) + \gamma^t) + w^i(k). \quad (6.26)$$

Using the ZF equalization [103] in (6.26), the t^{th} complex data symbol at k^{th} subcarrier can be estimated as:

$$\hat{s}^t(k) = \mu^t s^t(k) + \gamma^t + \left[(\mathbf{H}(k))^{\dagger} \mathbf{w}(k) \right]^t, \quad (6.27)$$

where $\left[(\mathbf{H}(k))^{\dagger} \mathbf{w}(k) \right]^t$ is t^{th} element of vector $(\mathbf{H}(k))^{\dagger} \mathbf{w}(k)$, $\mathbf{H}(k)$ is the frequency domain equivalent $N_R \times N_T$ MIMO channel at k^{th} subcarrier whose elements are $\tilde{h}_{i,t,k}$ and $\mathbf{w}(k)$ is the $N_R \times 1$ AWGN vector whose elements are $w^i(k)$. Thus, the resultant SDNR for k^{th} subcarrier can be given as:

$$SDNR^t(k) = \frac{P_s |\mu^t|^2}{\sigma_{\gamma^t}^2 + \left[\left((\mathbf{H}_k)^H \mathbf{H}_k \right)^{-1} \right]^t N_o} = \frac{\psi_k P_s |\mu^t|^2}{\psi_k \sigma_{\gamma^t}^2 + N_o}, \quad (6.28)$$

where ψ_k has a Chi-squared distribution with $2(N_R - N_T + 1)$ degrees of freedom and can be expressed as [103]

$$p_{\psi_k}(\psi_k) = \frac{\psi_k^{(N_R - N_T)} \Psi_k^{(N_R - N_T + 1)}}{\Psi_k^{(N_R - N_T + 1)} (N_R - N_T)!} e^{-\frac{\psi_k}{\Psi_k}}, \quad (6.29)$$

where $\Psi_k = \mathbb{E}[\psi_k]$. Therefore, SER of the data symbol for M -QAM at the k^{th} subcarrier can be expressed as [104]

$$f^t(k)(\psi_k) = 1 - \left(1 - \left(1 - \frac{1}{\sqrt{M}} \right) \text{erfc} \left(\sqrt{\frac{3SDNR^t(k)}{2(M-1)}} \right) \right)^2 \quad (6.30)$$

Therefore, the SER averaged across the fading channel can be expressed as:

$$SER^t(k) = \mathbb{E}[f^t(k)(\psi_k)] = \int_0^{\infty} f^t(k)(\psi_k) p_{\psi_k}(\psi_k) d\psi_k. \quad (6.31)$$

The analytical expression of SER in (6.31) is intractable and has to be evaluated numerically¹.

¹There are several softwares are available to compute the expression given in (6.31) such as MATLAB, Mathematica and MAPLE. Throughout this dissertation, the ‘integral’ function of MATLAB is used to evaluate this kind of expression.

Similar to (3.29) for large M and moderate to high SNR, (6.31)² can be bounded as:

$$SER^t(k) \approx A \left[1 - \frac{2}{\sqrt{\pi}} \sum_{f=0}^{\infty} \frac{(-1)^f B^{f+1/2}}{f!(2f+1)} \frac{\Gamma(N_R - N_T + f + 3/2)}{C^{N_R - N_T + f + 3/2}} \right. \\ \left. \times U\left(N_R - N_T + f + 3/2; N_R - N_T + 2; \frac{1}{C\Psi_k}\right) \right]. \quad (6.32)$$

As a special case of single-input-single-output (SISO) OFDM system with cascaded DPD+HPA architecture, the SER in (6.32) can be reduced to

$$SER(k) = \hat{A} \left[1 - \frac{1}{C\sqrt{\pi}} \sum_{f=0}^{\infty} \frac{(-1)^f \Gamma(f + 1/2)}{f!(2f+1)} \left(\frac{B}{C}\right)^{f+1/2} U\left(f + 3/2; 2; \frac{1}{C\Psi_k}\right) \right], \quad (6.33)$$

where $\hat{A} = \frac{2}{\Psi_k} \left(1 - \frac{1}{\sqrt{M}}\right)$. For linear MIMO-OFDM system, it is assumed that a linear HPA is present at each transmit antenna, i.e., $\mu^t = 1$ and $\gamma^t = 0$. Thus, SER for linear MIMO-OFDM system can be expressed as:

$$SER^t(k) = A \left[1 - 2\sqrt{\frac{\hat{B}}{\pi}} \Psi_k^{2(N_R - N_T) + 3} \Gamma(N_R - N_T + 3/2) {}_2F_1\left(1/2, N_R - N_T + 3/2; 3/2; -\hat{B}\Psi_k^2\right) \right]. \quad (6.34)$$

Using [106, 4.3.4], SER of linear SISO-OFDM system can be evaluated as:

$$SER(k) \approx \hat{A} \left(1 - \Psi_k \sqrt{\frac{\hat{B}}{\hat{B} + \Psi_k}} \right). \quad (6.35)$$

Hence, the average SER across all the transmitting branches and all the subcarriers can be evaluated as:

$$SER = \frac{1}{TN} \sum_{t=0}^{N_T-1} \sum_{k=0}^{N-1} SER^t(k). \quad (6.36)$$

Using (6.36), the end-to-end theoretical evaluation of the MIMO-OFDM system with and without a cascaded DPD+HPA architecture can be analyzed in terms of SER over the multi-path Rayleigh fading channel.

²The expression in (6.32) contains the infinite series. In order to evaluate this expression, the infinite series can be truncated to $e = 10$ terms with negligible loss in precision.

6.5 Simulation Results

In this section, the analytical and simulation results are presented to evaluate the end-to-end performance of the nonlinear MIMO-OFDM system in the presence of DPD. Specifically, the performance has been evaluated for the following systems: 1) SISO-OFDM system in AWGN channel in terms of SDR, EVM, NMSE and SER, and 2) MIMO-OFDM systems with a different number of transmit and receive antennas in multi-path Rayleigh fading channel in terms of SER. The simulations have also been compared with theoretical results to validate the analytically obtained expressions.

To characterize the nonlinear HPA, a traveling wave tube amplifier (TWTA) model [40] is used which can be given as:

$$B[\rho] = \frac{\theta_B \rho}{1 + \Phi_B \rho^2}, \quad \Psi[\rho] = \frac{\theta_\Psi \rho^2}{1 + \Phi_\Psi \rho^2}, \quad (6.37)$$

where $B[\rho]$ and $\Psi[\rho]$ are the normalized AM/AM and AM/PM characteristics, respectively. The values of θ_B , Φ_B , θ_Ψ , and Φ_Ψ are 1, 0.25, 0.26, and 0.25, respectively [126]. The coefficients of nonlinear polynomial model for HPA with 3rd order nonlinearity ($C = 2$) can be extracted from TWTA model by employing least-squares method [42].

In line with previous literature [28, 70], we assume that all the HPAs in MIMO system typically have the same behavioral characteristics. As a consequence, their corresponding DPDs will also have the same inverse characteristics. As mentioned earlier, the coefficients of DPD are extracted using the ILA approach at 1dB compression point. Thus, $\alpha_{2c-1}^t = \alpha_{2c-1}$ and $\beta_p^t = \beta_p \forall t \in [1, N_T]$ which implies $\mu^t = \mu$ and $\gamma^t = \gamma$ for transmitting branches. In the absence of DPD, the performance of a communication system impaired with nonlinear HPA is usually quantified by $\text{IBO} = \frac{A_{sat}^2}{P_{in}}$ where A_{sat} is the input saturation voltage of HPA [41].

6.5.1 SISO-OFDM System

In this section, the impact of cascaded DPD+HPA system on OFDM signal with $N = 2048$ subcarriers appended by $N_c = 512$ cyclic prefix is evaluated. Initially, we measure the impact of nonlinear distortion caused by HPA only and the performance of the cascaded system in terms of SDR and transmit EVM, i.e., in the absence of channel effects. Then the results are evaluated for the complete end-to-end performance of the aforementioned system in the presence of AWGN channel in terms of SER.

For the evaluation of SDR and EVM performance at the transmitter, we can ignore the AWGN effects by substituting $N_o = 0$ in (6.28). This leads to $\text{SDR} = P_s |\mu|^2 / \sigma_\gamma^2$ and correspondingly, $\text{EVM} = \sqrt{1/\text{SDR}}$ [20]. NMSE is also often used as a metric to quantify the in-band

Table 6.1: EVM (%) of the transmit signal in the presence of nonlinear HPA only.

IBO [dB]	1	5	9	11	13	17
EVM [%]	44.57	14.47	5.29	3.27	2.03	0.78
NMSE [dB]	-3.51	-8.4	-12.77	-14.85	-16.93	-21.08

Table 6.2: EVM (%) of the transmit signal in the presence of both DPD and HPA.

P	3	5	6
EVM [%]	3.65	2.04	0.02
NMSE [dB]	-14.38	-16.90	-36.99

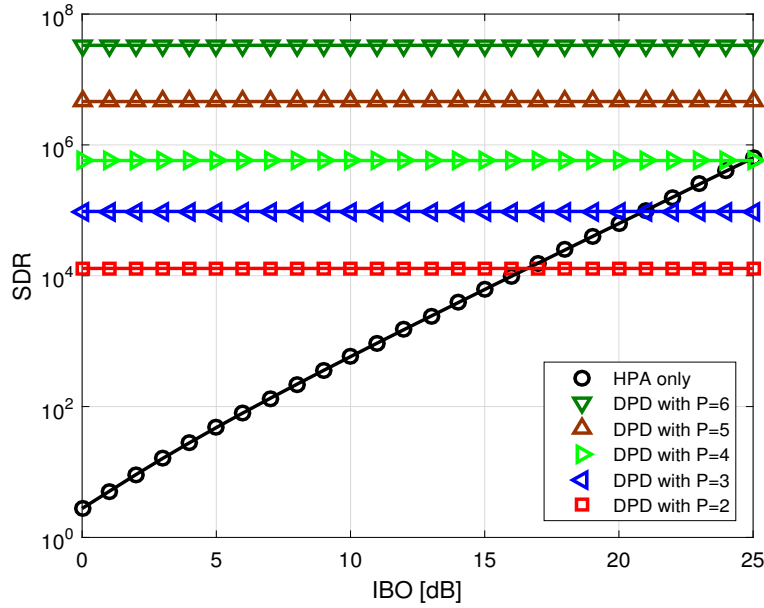


Figure 6.3: SDR performance of the transmit signal with different transmitter configuration.

distortion due to HPA [127]. It can be defined as $NMSE = \frac{\sum_{n=1}^N |y[n] - x[n]|^2}{\sum_{n=1}^N |x[n]|^2}$. Table 6.1 and 6.2 shows the EVM at the output of the HPA with and without DPD at the transmitter, respectively. As per 3GPP specifications TS 136.101 [111] and TS 136.104 [112], the maximum value of EVM at the transmitter for 16-QAM is 12.5%. It is quite obvious from Table 6.1 that the EVM at the transmitter can be kept under 3GPP specification by operating the nonlinear HPA with larger IBO. However, it reduces the power efficiency of the nonlinear HPA. On the other hand, the value of EVM at the transmitter is quite low in the presence of DPD as shown in Table 6.2. Here, it can also be observed that even the lower order DPD can be used to maintain the EVM under 3GPP specification.

Fig. 6.3 shows the impact of nonlinear distortion caused by the nonlinear HPA only and the residual distortion caused by the cascade chain of DPD and HPA on the in-band signal quality

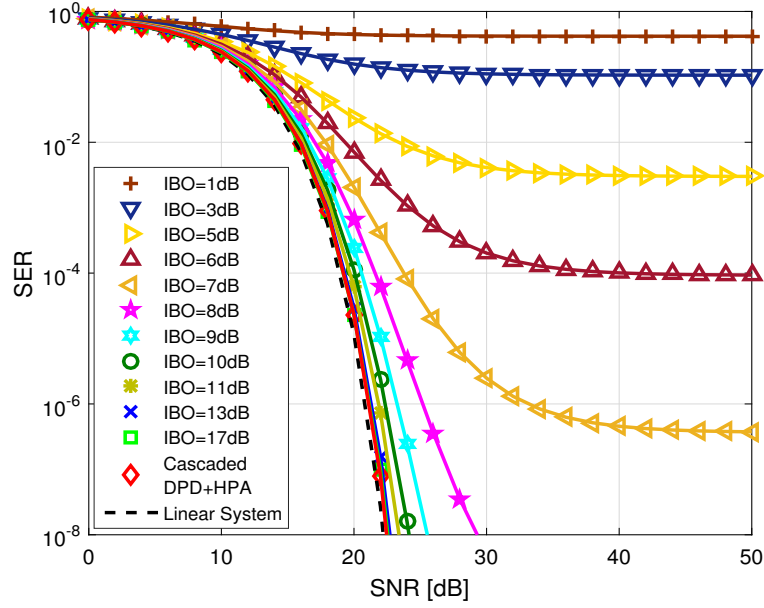


Figure 6.4: Analytical (solid lines) and simulation (marker points) results of average SER of SISO-OFDM system in the AWGN channel for 16 QAM.

in terms of SDR with different IBOs. Higher SDR leads to lower nonlinear distortion at the transmitter. It can be observed from Fig. 6.3 that the nonlinear distortion caused for the case of HPA only (i.e., in the absence of DPD) is reducing with an increase in the IBO. This result has been compared with SDR performance with cascaded DPD+HPA for the different number of nonlinearity order of the DPD. In the case of DPD, back-off needed for HPA can be reduced as DPD itself mitigates the nonlinear distortion caused by the HPA. Thus the HPA can be operated with higher efficiency. It has also been shown that the SDR performance improves with an increase in the order of DPD. Even for lowest order, i.e., $P = 2$, the nonlinear distortion is very low as compared to HPA nonlinearity with high back-off. This concludes that lower order DPD can be used to mitigate the in-band distortion. The excellent correlation between the analytical and simulation results also verifies the analytical expressions of complex attenuation factor and variance of the nonlinear noise.

Fig. 6.4 shows the SER performance as a function of AWGN SNR (P_s/N_o) of the nonlinear OFDM system in the presence of AWGN channel. The analytical and simulation results without DPD at different IBOs and in the presence of DPD with $P = 6$ coincide with each other which validates the proposed end-to-end analytical framework. It can also be observed that on increasing the IBO, SER performance in the presence of HPA only improves and approaches to a linear system, i.e., system with an ideal HPA. However, a high back-off is needed to attain the linear system performance which significantly reduces the power efficiency of the HPA. On the other hand, the overall performance of the nonlinear OFDM system with DPD have the

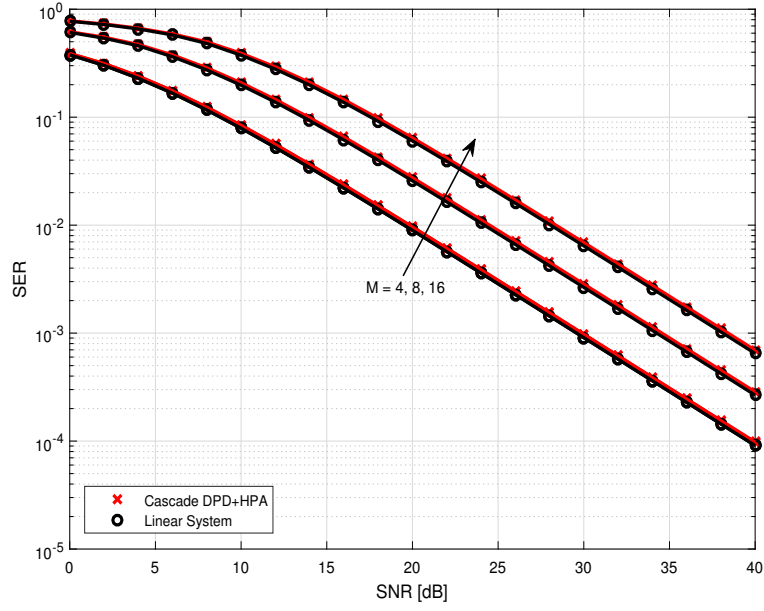


Figure 6.5: Analytical (solid lines) and simulation (marker points) results of average SER of SISO-OFDM system for different M-QAM.

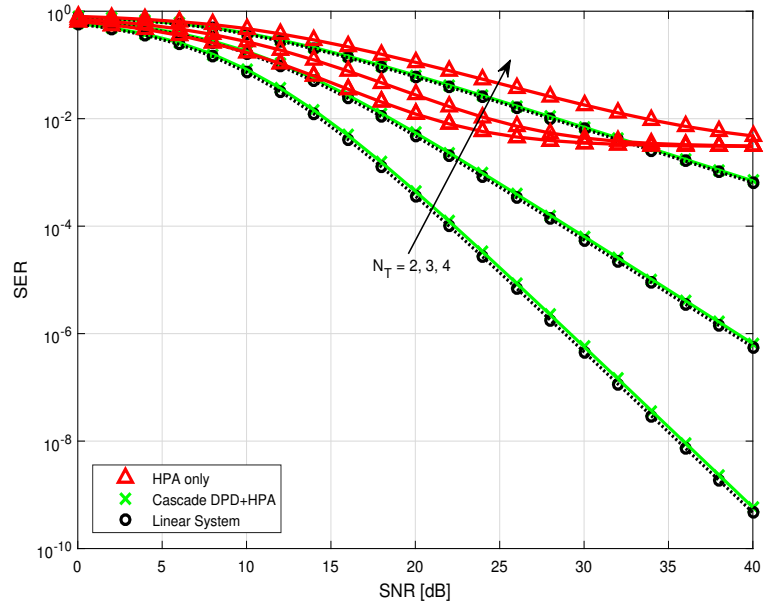


Figure 6.6: Analytical (solid lines) and simulation (marker points) results of average SER of different MIMO-OFDM systems for 16 QAM.

same performance as that of a linear system, without compromising the power efficiency of the HPA. Fig. 6.5 shows the SER performance of the nonlinear SISO-OFDM system for different M-QAM constellation size. It is a well-known fact that error performance deteriorates with increase in M-QAM constellation size which can also be observed from Fig. 6.5. However,

the performance of the cascaded system approaches to the linear system for every M-QAM constellation size. Furthermore, the analytical and simulation results coincide with each other for every constellation size which also strengthens our proposed framework.

6.5.2 MIMO-OFDM System

In this section, the SER performance of the MIMO-OFDM system with cascaded DPD+HPA chain at each transmitting branch has been evaluated in the presence of multi-path Rayleigh fading channel. The channel is represented by the six delay taps with uniform power delay profile [68]. The number of subcarriers and cyclic prefix length is same as given in subsection 6.5.1 in each OFDM signal. Each subcarrier contains complex data symbols from 16-QAM constellation.

Fig. 6.6 shows the average SER performance as a function of AWGN SNR for 4x2, 4x3 and 4x4 MIMO-OFDM in the presence of both DPD and HPA. As an ideal performance benchmark, the performance of the MIMO-OFDM system with a linear HPA is also plotted. The impact of diversity gain $(N_R - N_T + 1)$ on the performance of MIMO-OFDM systems can also be observed from Fig. 6.6. The excellent agreement between the analytical and simulation results validates the proposed analysis for cascaded DPD+ HPA architecture. It can be observed from Fig. 6.6 that there is no effect of diversity on the error performance due to the dominance of nonlinear noise in case of non-linear HPA. However, the residual noise is very low in case of DPD+HPA. As a consequence, the impact of diversity can be observed in the error performance. It is worth mentioning that the SER performance of the nonlinear MIMO-OFDM system with DPD is slightly lower than the linear system. This is due to the limitation on the maximum linearization performance that can be achieved by conventional ILA DPD approach. Hence, the output of cascaded DPD+HPA architecture will have some residual noise making its performance slightly inferior to that of a complete linear system.

6.6 Conclusion

In this chapter, we analyzed the impact of DPD on the in-band signal quality of a nonlinear MIMO-OFDM system for a multi-path Rayleigh fading MIMO channel. The nonlinear behavior of HPA and DPD have been modeled using a nonlinear polynomial model. The joint polynomial model is proposed to characterize the impact of cascaded DPD+HPA architecture. It has been shown that there is residual distortion at the output of this cascaded system which can be canonically decoupled in terms of the multiplicative factor and additive zero-mean Gaussian residual noise. The closed-form expression of the multiplicative factor and variance of the

residual noise are derived. Using these expressions, a complete analytical methodology is developed to obtain the closed-form expression of SER of MIMO-OFDM systems with cascaded DPA+HPA architecture. It has also been shown that the cascaded DPD+HPA linearizes the transmit output and improves the performance of nonlinear wireless systems. Thus, we can say DPD+HPA is a best possible solution to the problem of non-linear HPA. However, there are other research issues are related to both HPA and DPD such as accurate model extraction, complexity, bandwidth requirement, etc. which are not addressed this chapter. Due to these issues, the performance of cascaded DPD+HPA architecture can be degraded; however it does not change the expressions obtained in this chapter. These problem are the part of future scope. This framework can easily be utilized in various wireless standards and will be useful for a communication engineer to design a link budget for MIMO-OFDM based wireless systems without performing extensive simulations or tedious experiments.

Conclusion and Future Work

Advancement in seamless wireless connectivity has led to an upsurge in data traffic over a wireless network which necessitates the efficient utilization of available resources. To support such data traffic, wireless communication systems need to deploy multi-antenna and multi-carrier modulation techniques such as MIMO-OFDM. However, MIMO-OFDM is still not enough as a standalone to fulfill the ever increasing demand for high data rate transmission. To satisfy this demand, the new features of LTE-A standard such as CA and MU-MIMO can be used together with OFDM which significantly boosts the performance of the wireless communication systems. However due to inherent nonlinear behavior of the HPA, the performance of MIMO-OFDM based system degrades considerably. To ensure the linear operation, HPA can be forced to operate with high IBO; however, it compromises its power efficiency. DPD is another solution to mitigate the nonlinear distortion without affecting its power efficiency. The nonlinear distortion caused by HPA becomes more severe in the presence of MU and CA-OFDM signals. Furthermore, the non-ideal oscillators cause phase noise which disrupts the orthogonality among subcarriers of the OFDM signal and leads to ICI which also degrades the performance of the MIMO-OFDM system. These impairments severely limit the gains provided by the MIMO-OFDM systems as compared to the linear systems.

In this dissertation, it has been shown that the hardware impairments create challenges while evaluating the performance of MIMO-OFDM based system which are not adequately addressed in the literature. These challenges and research gaps are summarized as:

- The conventional behavioral models are insufficient to characterize the nonlinear distortion caused by the HPA when driven by multi-band CA-OFDM signal.
- The nonlinear distortion depends on the number of users and affects the performance of MU-MIMO-OFDM system different as compared to the conventional MIMO-OFDM system.

- The complete theoretical analysis of the joint CA and MU-MIMO techniques along with OFDM signals in the presence of nonlinear HPA has not been investigated.
- The phase noise caused by the non-ideal oscillator makes the frequency domain analysis of the MU-MIMO-OFDM system more complex. As a consequence, it is difficult to theoretically analyze the impact of phase noise on MU-MIMO-OFDM system.
- There is no behavioral model present in the literature to characterize the cascaded DPD+HPA architecture. Thus, it is not feasible to mathematically analyze the end-to-end performance of the MIMO-OFDM systems in the presence of both DPD and HPA.

7.1 Summary of the Contributions

The primary goal of this dissertation has been the development of the analytical framework to evaluate the end-to-end performance of the MIMO-OFDM based systems in the presence of nonlinear transceivers. First, the impact of nonlinear HPA is investigated on the performance of the CA and MU techniques both separately and together for the MIMO-OFDM system followed by evaluation of the performance of MU-MIMO-OFDM system in the presence of both nonlinear HPA and non-ideal oscillators. Finally, the mathematical representation of the cascaded DPD+HPA architecture is proposed to evaluate the performance of the nonlinear multi-carrier wideband communication systems in the presence of the DPD. Specifically, the contribution of this dissertation is concluded as follows:

- The performance of CA-MIMO-OFDM system in the presence of nonlinear HPA for a multi-path Rayleigh fading MIMO channel is analytically evaluated. The nonlinear distortions at the output of the HPA are canonically characterized in terms of complex attenuation factor and additive zero-mean Gaussian nonlinear noise. Their generalized expressions are derived that are applicable to any number of aggregated bands with any nonlinearity and memory depth. From the expressions, a complete analytical approach is presented to obtain the SER and EVM for each subcarrier of the CA-MIMO-OFDM systems. It was also observed that the nonlinear distortion caused by multi-band HPA increases with an increase in the number of aggregated bands which diminishes the gain due to diversity and SNR.
- The impact of the nonlinear transmitter on the performance of DB MU-MIMO-OFDM system is theoretically investigated in terms of SER and overall capacity. The analytical expressions for nonlinear interference caused due to HPA nonlinearity and SINR are derived. It was observed that the nonlinear interference is a function of the number

of users and number of aggregated bands which deteriorates the performance of the nonlinear DB MU-MIMO-OFDM system by introducing an irreducible error floor. It was also observed a trade-off between the improvement in capacity and operating HPAs with higher IBOs. Although for nonlinear MU-MIMO-OFDM system capacity scales moderately with the number of users and bands, the gains are severely limited when compared to the linear system. Thus, the transmitter nonlinearity not only compromises the reliability but also restricts the average capacity of the system.

- The analytical methodology is proposed to study the joint impact of non-ideal oscillators and nonlinear HPAs on the performance of the MU-MIMO-OFDM system. The impact has been investigated by deriving the closed-form expressions for SER and capacity of the MU-MIMO-OFDM system in the presence of both phase noise and nonlinear distortion. It has been shown that by utilizing these derived expressions, it is possible to evaluate the performance of MU-MIMO-OFDM system in the presence or absence of any of these impairments. The joint impact of phase noise and nonlinear distortion significantly deteriorate the performance of the MU-MIMO-OFDM system as compared to the ideal system. There are three trade-offs associated with the MU-MIMO-OFDM system in the presence of hardware impairments: 1) Trade-off between MUI elimination and transmit diversity, 2) Trade-off between operating HPA with high back-off and power efficiency, 3) Trade-off between phase noise tolerance and the number of users to achieve maximum capacity. These trade-offs severely restrict the performance gains of the MU-MIMO-OFDM system.
- The impact of DPD on the in-band signal quality of the nonlinear MIMO-OFDM system is evaluated in the presence of the multi-path Rayleigh fading MIMO channel. The JNP model is proposed to represent the characteristics of the cascaded DPD+HPA architecture. It was shown that the received symbol after the MIMO-OFDM demodulation consists of residual noise and a multiplicative factor. The closed-form expression of the multiplicative factor and variance of the residual noise are derived. Using these expressions, a complete analytical methodology is developed to obtain the closed-form expression of SER of MIMO-OFDM systems with cascaded DPA+HPA architecture. It was observed that even the lower order DPD can be used to mitigate the in-band distortion which in turn helps in improving the system reliability.

The above proposed frameworks can easily be utilized in various wireless standards and will be useful for a communication engineer to design a link budget for MIMO-OFDM based wireless systems without performing extensive simulations or tedious experiments.

7.2 Future Work

There are many possible directions in which the work articulated in this dissertation can be extended. Some of the future research directions are underlined as:

- Apart from nonlinear distortion and phase noise, there are several other transceiver impairments present in the wireless communication systems such as MIMO crosstalk and I/Q imbalance. These impairments also deteriorate the performance of the wireless communication system. Hence, it is imperative to take into account the impact of all impairments while analytically evaluating the performance of CA MU-MIMO-OFDM system.
- In this dissertation, a memoryless JNP model is proposed to characterize the cascaded DPD+HPA architecture. However, it does not consider the memory effects. Thus, an enhanced JNP model can be developed which considers the memory effects of cascaded DPD+HPA architecture. Furthermore, the expression of the coefficients of the proposed JNP model are specific to a certain nonlinearity order of HPA and DPD. Therefore, the generalized expression of the coefficient of the JNP model can also be derived which can be applied to any nonlinearity order and memory depth of both DPD and HPA. Furthermore, this model can also be extended for cascaded DPD+Crosstalk+HPA architecture.
- In-band full-duplex (IBFD) system has recently garnered significant attention in academia and industry because it has the potential of being able to increase spectral efficiency without the need for additional frequency resources. In the IBFD system, there is simultaneous transmission and reception of data signals within the same frequency band. On utilizing the benefits of MU-MIMO and IBFD together, the spectral efficiency of the wireless communication system can be improved significantly. This merger of MU-MIMO and IBFD system can be commonly referred as MU-IBFD system. The performance of this system can be analyzed in the presence of the non-ideal devices. Furthermore, the joint linearization and interference cancellation technique can also be developed to simultaneously mitigate the nonlinear distortion (by HPA), the multi-user interference (due to MU-MIMO transmission), and the self-interference (due to IBFD transmission).

Appendix A

Decomposition of the received complex data symbol

Multiply (3.12) $s_b^{t*}(k)$, taking the expectation of the resultant term with an assumption that $\mu_b(k)$ is an unknown parameter. Thus, we have

$$\mathbb{E}[\tilde{s}_b^t(k)s_b^{t*}(k)] = \mu_b(k)P_s/B. \quad (\text{A.36})$$

Substituting $\tilde{s}_b^t(k)$ from (3.12) and $s_b^{t*}(k)$ by its IDFT, we obtain

$$\begin{aligned} \mathbb{E}[\tilde{s}_b^t(k)s_b^{t*}(k)] = & \mathbb{E} \left[\sum_{n=0}^{N-1} \sum_{p_1=1}^P \sum_{p_2=1}^{p_1} \cdots \sum_{p_B=1}^{p_{B-1}} \sum_{g=-G}^G \alpha_{p_1,p_2,\dots,p_B,k,g} x_b^t[n+g] |x_1^t[n]|^{p_1-p_2} \dots \right. \\ & \left. \times |x_B^t[n]|^{p_B-1} e^{-j2\pi k(n+g)/N} \sum_{u=0}^{N-1} x_b^{t*}[u] e^{j2\pi uk/N} \right]. \end{aligned} \quad (\text{A.37})$$

As stated earlier, $x_b^t[n]$ can be approximated to Gaussian random process using CLT with zero mean and variance $P_{in} = P_s/NB$. Hence, any function of $x_b^t[n_1]$ and $x_b^t[n_2]$ with $n_1 \neq n_2$ are jointly Gaussian. Moreover, two different components of IDFT of i.i.d. random sequence are uncorrelated [68]. Hence, $x_b^t[n_1]$ and $x_b^t[n_2]$ are uncorrelated. Since, $x_b^t[n_1]$ and $x_b^t[n_2]$ are jointly Gaussian and uncorrelated, therefore they are independent as well. Furthermore, $|x_b^t[n]|$ is Rayleigh distributed random variable whose central moment is known. In (A.37), if $n+g \neq u$

the expected value is zero, therefore we have

$$\begin{aligned} \mathbb{E}[\tilde{s}_b^t(k)s_b^{t*}(k)] &= \sum_{n=0}^{N-1} \sum_{p_1=1}^P \sum_{p_2=1}^{p_1} \cdots \sum_{p_B=1}^{p_{B-1}} \sum_{g=-G}^G \alpha_{p_1,p_2,\dots,p_B,k,g} \mathbb{E} \left[|x_b^t[n+g]|^2 |x_1^t[n]|^{p_1-p_2} \cdots \right. \\ &\quad \left. \times |x_B^t[n]|^{p_B-1} \right]. \end{aligned} \quad (\text{A.38})$$

After some manipulations and using (A.36) and (A.38), $\mu_b(k)$ can be expressed as:

$$\begin{aligned} \mu_b(k) &= \sum_{p_1=1}^P \sum_{p_2=1}^{p_1} \cdots \sum_{p_B=1}^{p_{B-1}} (P_{in})^{\frac{p_1-1}{2}} \left(\alpha_{p_1,p_2,\dots,p_B,k,0} \Gamma \left(1 + \frac{p_1-p_2}{2} \right) \cdots \right. \\ &\quad \times \Gamma \left(1 + \frac{p_b-p_{b+1}+2}{2} \right) \cdots \Gamma \left(1 + \frac{p_B-1}{2} \right) + \sum_{\substack{g=-G \\ g \neq 0}}^G \alpha_{p_1,p_2,\dots,p_B,k,g} \Gamma(2) \\ &\quad \left. \times \Gamma \left(1 + \frac{p_1-p_2}{2} \right) \cdots \Gamma \left(1 + \frac{p_B-1}{2} \right) \right). \end{aligned} \quad (\text{A.39})$$

As a result, $\tilde{s}_b^t(k)$ can be expressed as:

$$\tilde{s}_b^t(k) = \mu_b(k)s_b^t(k) + \phi_b^t(k), \quad (\text{A.40})$$

where $\phi_b^t(k)$ is a nonlinear noise whose expression is given in (3.18).

Appendix B

Mean of nonlinear noise

The mean of nonlinear noise $\phi_b^t(k)$ can be expressed as:

$$\mathbb{E}[\phi_b^t(k)] = \mathbb{E}[\tilde{s}_b^t(k)] - \mu_b(k)\mathbb{E}[s_b^t(k)]. \quad (\text{B.1})$$

Since, $s_b^t(k)$ has zero mean, i.e., $\mathbb{E}[s_b^t(k)] = 0$. Thus, we have

$$\mathbb{E}[\phi_b^t(k)] = \mathbb{E}[\tilde{s}_b^t(k)]. \quad (\text{B.2})$$

Using (3.12) and replacing $x_b^t[n+g]$ by its IDFT, we have

$$\begin{aligned} \tilde{s}_b^t(k) &= \sum_{n=0}^{N-1} \sum_{p_1=1}^P \sum_{p_2=1}^{p_1} \cdots \sum_{p_B=1}^{p_{B-1}} \sum_{m=0}^{M-1} \sum_{g=-G}^G \alpha_{p_1, p_2, \dots, p_B, k, g} |x_1^t[n]|^{p_1-p_2} \cdots |x_B^t[n]|^{p_B-1} \\ &\quad \times e^{-j2\pi k(n+g)/N} \frac{1}{N} \sum_{u=0}^{N-1} s_b^t(u) e^{j2\pi u(n+g)/N}. \end{aligned} \quad (\text{B.3})$$

On substituting IDFT of $s_b^t(u)$ and taking the expectation of $\tilde{s}_b^t(k)$, we have

$$\begin{aligned} \mathbb{E}[\tilde{s}_b^t(k)] &= \sum_{n=0}^{N-1} \sum_{p_1=1}^P \sum_{p_2=1}^{p_1} \cdots \sum_{p_B=1}^{p_{B-1}} \sum_{g=-G}^G \sum_{u=0}^{N-1} \alpha_{p_1, p_2, \dots, p_B, k, g} \frac{1}{N} \sum_{v=0}^{N-1} \mathbb{E} \left[|x_b^t[v]| |x_1^t[n]|^{p_1-p_2} \cdots \right. \\ &\quad \left. \times |x_B^t[n]|^{p_B-1} \right] e^{-j2\pi k(n+g)/N} e^{j2\pi u(n+g)/N} e^{-j2\pi uv/N}. \end{aligned} \quad (\text{B.4})$$

If $v \neq n$, then expectation will be zero, thus we have

$$\begin{aligned} E[\tilde{s}_b^t(k)] &= \sum_{n=0}^{N-1} \sum_{p_1=1}^P \sum_{p_2=1}^{p_1} \cdots \sum_{p_B=1}^{p_{B-1}} \sum_{g=-G}^G \sum_{u=0}^{N-1} \alpha_{p_1, p_2, \dots, p_B, k, g} \frac{1}{N} E \left[|x_b^t[n]|^{p_1-p_2} \cdots \right. \\ &\quad \times \left. |x_B^t[n]|^{p_B-1} \right] e^{-j2\pi k(n+g)/N} e^{j2\pi ug/N}. \end{aligned} \quad (\text{B.5})$$

We know that all the OFDM signals $x_b^t[n]$ independent to each other, we can rewrite the above equation as

$$\begin{aligned} E[\tilde{s}_b^t(k)] &= \sum_{n=0}^{N-1} \sum_{p_1=1}^P \sum_{p_2=1}^{p_1} \cdots \sum_{p_B=1}^{p_{B-1}} \sum_{g=-G}^G \sum_{u=0}^{N-1} \alpha_{p_1, p_2, \dots, p_B, k, g} \frac{1}{N} E \left[|x_1^t[n]|^{p_1-p_2} \right] \cdots \\ &\quad \times E \left[|x_b^t[n]|^{p_b-p_{b+1}} \right] \cdots E \left[|x_B^t[n]|^{p_B-1} \right] e^{-j2\pi k(n+g)/N} e^{j2\pi ug/N}. \end{aligned} \quad (\text{B.6})$$

Since, $x_b^t[n]$ is circularly symmetric complex-valued Gaussian random variable and $|x_b^t[n]|^{\hat{p}}$ is a random positive number, where $\hat{p} = p_b - p_{b+1}$. Therefore, the new random variable $x_b^t[n]|x_b^t[n]|^{\hat{p}}$ is also circularly symmetric and its mean is zero. To prove this mathematically, we have $z = \sqrt{\text{real}(x_b^t[n])^2 + \text{imag}(x_b^t[n])^2}$ and $\theta = \tan^{-1} \frac{\text{imag}(x_b^t[n])}{\text{real}(x_b^t[n])}$ where z and θ are amplitude and phase of $x_b^t[n]$ and are independent with Rayleigh and uniform distribution, respectively. Their joint distribution is

$$f_{z,\theta}(z, \theta) = \frac{z}{2\pi\sigma^2} e^{-\frac{z^2}{2\sigma^2}}. \quad (\text{B.7})$$

where σ^2 is a common variance of $\text{real}(x_b^t[n])$ and $\text{imag}(x_b^t[n])$. Thus, the expectation of $x_b^t[n]|x_b^t[n]|^{\hat{p}}$ can be expressed as:

$$E \left[|x_b^t[n]|^q \right] = \int_{-\pi}^{\pi} \int_0^{\infty} z e^{j\theta} z^p f_{z,\theta}(z, \theta) dz d\theta = \int_0^{\infty} \frac{z^{q+1}}{2\pi\sigma^2} e^{-\frac{z^2}{2\sigma^2}} dr \int_{-\pi}^{\pi} e^{j\theta} d\theta = 0. \quad (\text{B.8})$$

Using (B.2), (B.6) and (B.8), it is proved that nonlinear noise has zero mean, i.e.,

$$E[\phi_b^t(k)] = E[\tilde{s}_b^t(k)] = 0. \quad (\text{B.9})$$

Appendix C

Variance of nonlinear noise

The variance of nonlinear noise can be given as

$$\sigma_{\phi_b^t(k)}^2 = \mathbb{E} \left[|\phi_b^t(k)|^2 \right] = \mathbb{E} \left[|\tilde{s}_b^t(k)|^2 \right] - |\mu_b(k)|^2 P_s/B. \quad (\text{C.1})$$

Using (B.3) and IDFT of $s_b^t(u)$, we have

$$\begin{aligned} \mathbb{E} \left[|\tilde{s}_b^t(k)|^2 \right] &= \frac{1}{N^2} \sum_{n=0}^{N-1} \sum_{p_1=1}^P \sum_{p_2=1}^{p_1} \cdots \sum_{p_B=1}^{p_{B-1}} \sum_{g=-G}^G \sum_{u=0}^{N-1} \sum_{v=0}^{N-1} \sum_{n'=0}^{N-1} \sum_{p'_1=1}^P \sum_{p'_2=1}^{p'_1} \cdots \sum_{p'_B=1}^{p'_{B-1}} \sum_{g'=-G}^G \sum_{u'=0}^{N-1} \\ &\quad \sum_{v'=0}^{N-1} \alpha_{p_1, p_2, \dots, p_B, k, g} \alpha_{p'_1, p'_2, \dots, p'_B, k, g'}^* e^{-j2\pi k(n+g)/N} e^{j2\pi u(n+g)/N} e^{-j2\pi uv/N} \\ &\quad \times e^{j2\pi k(n'+g')/N} e^{j2\pi u'v'/N} e^{-j2\pi u'(n'+g')/N} \mathbb{E} \left[|x_b^t[v]| |x_1^t[n]|^{p_1-p_2} \dots \right. \\ &\quad \left. \times |x_B^t[n]|^{p_B-1} |x_b^t[v']|^* |x_1^t[n']|^{p'_1-p'_2} \dots |x_B^t[n']|^{p'_B-1} \right]. \end{aligned} \quad (\text{C.2})$$

If $v \neq v'$, then the expectation value leads to zero, thus we have

$$\begin{aligned} \mathbb{E} \left[|\tilde{s}_b^t(k)|^2 \right] &= \frac{1}{N^2} \sum_{p_1=1}^P \sum_{p_2=1}^{p_1} \cdots \sum_{p_B=1}^{p_{B-1}} \sum_{g=-G}^G \sum_{p'_1=1}^P \sum_{p'_2=1}^{p'_1} \cdots \sum_{p'_B=1}^{p'_{B-1}} \sum_{g'=-G}^G \alpha_{p_1, p_2, \dots, p_B, k, g} \\ &\quad \alpha_{p'_1, p'_2, \dots, p'_B, k, g'}^* \rho_{p_1, p_2, \dots, p_B, g, p'_1, p'_2, \dots, p'_B, g', k}^t \end{aligned} \quad (\text{C.3})$$

where

$$\begin{aligned}
 \rho_{p_1, p_2, \dots, p_B, g, p'_1, p'_2, \dots, p'_B, g', k}^t &= \sum_{n=0}^{N-1} \sum_{u=0}^{N-1} \sum_{v=0}^{N-1} \sum_{n'=0}^{N-1} \sum_{u'=0}^{N-1} E \left[|x_b^t[v]|^2 |x_1^t[n]|^{p_1-p_2} \dots |x_B^t[n]|^{p_B-1} \right. \\
 &\quad \times |x_1^t[n']|^{p'_1-p'_2} \dots |x_B^t[n']|^{p'_B-1} \left. \right] e^{j2\pi u(n+g)/N} e^{-j2\pi u'(n'+g')/N} \\
 &\quad \times e^{-j2\pi v(u-u')/N} e^{-j2\pi k(n+g-n'-g')/N} \\
 &= N^3 (\rho_1^t + \rho_2^t + \rho_3^t + \rho_4^t + \rho_5^t). \tag{C.4}
 \end{aligned}$$

There are five possible cases depending upon the value of v , n and n' .

If $v = n = n'$, we have

$$\begin{aligned}
 \rho_1^t &= \delta(g) \delta(g') \Gamma \left(1 + \frac{p_1 - p_2 + p'_1 - p'_2}{2} \right) \dots \Gamma \left(1 + \frac{p_b - p_{b+1} + p'_b - p'_{b+1} + 2}{2} \right) \\
 &\quad \dots \Gamma \left(1 + \frac{p_B + p'_B - 2}{2} \right) (P_{in})^{\frac{p_1 + p'_1}{2}}. \tag{C.5}
 \end{aligned}$$

If $v = n \neq n'$, we have

$$\begin{aligned}
 \rho_2^t &= \delta(g) (1 - \delta(g')) \Gamma \left(1 + \frac{p_1 - p_2}{2} \right) \Gamma \left(1 + \frac{p'_1 - p'_2}{2} \right) \dots \Gamma \left(1 + \frac{p_b - p_{b+1} + 2}{2} \right) \\
 &\quad \times \Gamma \left(1 + \frac{p'_b - p'_{b+1}}{2} \right) \dots \Gamma \left(1 + \frac{p_B - 1}{2} \right) \Gamma \left(1 + \frac{p'_B - 1}{2} \right) (P_{in})^{\frac{p_1 + p'_1}{2}}. \tag{C.6}
 \end{aligned}$$

If $v = n' \neq n$, we have

$$\begin{aligned}
 \rho_3^t &= \delta(g') (1 - \delta(g)) \Gamma \left(1 + \frac{p_1 - p_2}{2} \right) \Gamma \left(1 + \frac{p'_1 - p'_2}{2} \right) \dots \Gamma \left(1 + \frac{p_b - p_{b+1}}{2} \right) \\
 &\quad \times \Gamma \left(1 + \frac{p'_b - p'_{b+1} + 2}{2} \right) \dots \Gamma \left(1 + \frac{p_B - 1}{2} \right) \Gamma \left(1 + \frac{p'_B - 1}{2} \right) (P_{in})^{\frac{p_1 + p'_1}{2}}. \tag{C.7}
 \end{aligned}$$

If $v \neq n = n'$, we have

$$\begin{aligned}
 \rho_4^t &= (1 - \delta(g)) (1 - \delta(g')) \delta(g - g') \Gamma(2) \Gamma \left(1 + \frac{p_1 - p_2 + p'_1 - p'_2}{2} \right) \dots \\
 &\quad \times \Gamma \left(1 + \frac{p_b - p_{b+1} + p'_b - p'_{b+1}}{2} \right) \dots \Gamma \left(1 + \frac{p_B + p'_B - 2}{2} \right) (P_{in})^{\frac{p_1 + p'_1}{2}}. \tag{C.8}
 \end{aligned}$$

If $v \neq n \neq n'$, we have

$$\begin{aligned}
 \rho_5^t = & (1 - \delta(g)) (1 - \delta(g')) (1 - \delta(g - g')) \Gamma\left(1 + \frac{p_1 - p_2}{2}\right) \Gamma\left(1 + \frac{p'_1 - p'_2}{2}\right) \dots \\
 & \times \Gamma\left(1 + \frac{p_b - p_{b+1}}{2}\right) \Gamma\left(1 + \frac{p'_b - p'_{b+1}}{2}\right) \dots \Gamma\left(1 + \frac{p_B - 1}{2}\right) \Gamma\left(1 + \frac{p'_B - 1}{2}\right) \\
 & \times \Gamma(2) (P_{in})^{\frac{p_1 + p'_1}{2}}. \tag{C.9}
 \end{aligned}$$

Statistical properties of precoded nonlinear distortion vector

We will find the value of $\tilde{\Phi}_1(k)$ and then derive its statistical properties. We express the nonlinear distortion vector $\Phi_1(k)$ as:

$$\Phi_1(k) = \mathbf{V}_u(k) \tilde{\Phi}_1(k). \quad (\text{D.1})$$

On multiplying $\mathbf{V}_u(k)^H$ on both sides of (D.1), we obtain

$$\mathbf{V}_u(k)^H \Phi_1(k) = \mathbf{V}_u(k)^H \mathbf{V}_u(k) \tilde{\Phi}_1(k). \quad (\text{D.2})$$

On substituting the value of $\mathbf{V}_u(k)$ from (4.14) on the right side of (D.2), we get

$$\mathbf{V}_u(k)^H \Phi_1(k) = \mathbf{F}_u(k)^H \mathbf{E}_u(k)^H \mathbf{E}_u(k) \mathbf{F}_u(k) \tilde{\Phi}_1(k). \quad (\text{D.3})$$

Since, $\mathbf{E}_u(k)$ is the set of orthonormal basis, then $\mathbf{E}_u(k)^H \mathbf{E}_u(k) = \mathbf{I}_{A_u}$, where $\mathbf{I}_N$ is the identity matrix of order N. Furthermore, $\mathbf{F}_u(k)$ is an unitary matrix which implies $\mathbf{F}_u(k)^H \mathbf{F}_u(k) = \mathbf{I}_{Q_u}$. Therefore, (D.3) can be rewritten as:

$$\tilde{\Phi}_1(k) = \mathbf{V}_u(k)^H \Phi_1(k). \quad (\text{D.4})$$

It can be observed from (D.4) that $\tilde{\Phi}_1(k)$ is a $Q_u \times 1$ precoded nonlinear distortion vector whose each element is the weighted sum of the Gaussian random variable which implies that each element of $\tilde{\Phi}_1(k)$ is also a Gaussian random variable. The mean of $\tilde{\Phi}_1(k)$ can be computed as:

$$\mathbb{E} [\tilde{\Phi}_1(k)] = \mathbb{E} [\mathbf{V}_u(k)^H \Phi_1(k)] = \mathbf{V}_u(k)^H \mathbb{E} [\Phi_1(k)] = \mathbf{0}, \quad (\text{D.5})$$

and its variance can be given as:

$$\begin{aligned} \mathbb{E} [\tilde{\mathbf{\Phi}}_1(k) \tilde{\mathbf{\Phi}}_1(k)^H] &= \mathbb{E} [\mathbf{V}_u(k)^H \mathbf{\Phi}_1(k) \mathbf{\Phi}_1(k)^H \mathbf{V}_u(k)] = \mathbf{V}_u(k)^H \mathbb{E} [\mathbf{\Phi}_1(k) \mathbf{\Phi}_1(k)^H] \mathbf{V}_u(k) \\ &= \mathbf{V}_u(k)^H N \sigma_{\phi_1^z(k)}^2 \mathbf{I}_{N_T} \mathbf{V}_u(k) = N \sigma_{\phi_1^z(k)}^2 \mathbf{V}_u(k)^H \mathbf{V}_u(k) = N \sigma_{\phi_1^z(k)}^2 \mathbf{I}_{Q_u}. \quad (\text{D.6}) \end{aligned}$$

Therefore, $\tilde{\mathbf{\Phi}}_1(k)$ is a precoded nonlinear distortion vector whose elements are $\mathcal{CN}(0, N \sigma_{\phi_1^z(k)}^2)$.

Appendix E

Mean and variance of $\zeta(0)$

Using the fact that $\vartheta[n] \sim \mathcal{N}(0, \sigma_\vartheta^2)$, the mean of $\zeta(0)$ can be computed as:

$$\mathbb{E}[\zeta(0)] = \mathbb{E}\left[\frac{1}{N} \sum_{n=0}^{N-1} e^{j\vartheta[n]}\right] = \frac{1}{N} \sum_{n=0}^{N-1} \mathbb{E}\left[e^{j\vartheta[n]}\right] = \frac{1}{N} \sum_{n=0}^{N-1} \int_{-\infty}^{\infty} e^{j\vartheta[n]} \frac{1}{\sqrt{2\pi}\sigma_\vartheta} e^{-\frac{\vartheta^2[n]}{2\sigma_\vartheta^2}} d\vartheta[n] \quad (\text{E.1})$$

$$= \frac{1}{N} \sum_{n=0}^{N-1} e^{-\sigma_\vartheta^2/2} \underbrace{\int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}\sigma_\vartheta} e^{-\frac{(\vartheta[n]-j\sigma_\vartheta^2)^2}{2\sigma_\vartheta^2}} d\vartheta[n]}_{=1} = e^{-\sigma_\vartheta^2/2} \quad (\text{E.2})$$

The variance of $\zeta(0)$ can be expressed as:

$$\text{var}[\zeta(0)] = \frac{1}{N^2} \text{var}\left[\sum_{n=0}^{N-1} e^{j\vartheta[n]}\right] = \frac{1}{N^2} \left(\sum_{n=0}^{N-1} \text{var}\left[e^{j\vartheta[n]}\right] + \sum_{n \neq n'} \text{Cov}\left(e^{j\vartheta[n]}, e^{j\vartheta[n']}\right) \right), \quad (\text{E.3})$$

where $\text{Cov}(\cdot, \cdot)$ is the covariance which can be expressed as:

$$\text{Cov}\left(e^{j\vartheta[n]}, e^{j\vartheta[n']}\right) = \mathbb{E}\left[e^{j\vartheta[n]} e^{-j\vartheta[n']}\right] - \mathbb{E}\left[e^{j\vartheta[n]}\right] \mathbb{E}\left[e^{-j\vartheta[n']}\right] = \mathbb{E}\left[e^{j(\vartheta[n]-\vartheta[n'])}\right] - e^{-\sigma_\vartheta^2} \quad (\text{E.4})$$

Since $\vartheta[n]$ is a Gaussian random variable, then $\vartheta[n] - \vartheta[n']$ is also Gaussian random variable with zero mean and $2\sigma_\vartheta^2$. Thus, $\mathbb{E}\left[e^{j(\vartheta[n]-\vartheta[n'])}\right] = e^{-\sigma_\vartheta^2}$ which implies $\text{Cov}\left(e^{j\vartheta[n]}, e^{j\vartheta[n']}\right) =$

0. Therefore, the variance of $\zeta(0)$ can be written as:

$$\text{var}[\zeta(0)] = \frac{1}{N^2} \sum_{n=0}^{N-1} \text{var}[e^{j\vartheta[n]}] = \frac{1}{N^2} \sum_{n=0}^{N-1} \left(\mathbb{E}[|e^{j\vartheta[n]}|^2] - |\mathbb{E}[e^{j\vartheta[n]}]|^2 \right) = \frac{1}{N} \left(1 - e^{-\sigma_{\vartheta}^2} \right) \quad (\text{E.5})$$

Appendix F

Mean and variance of $\zeta(k' - k)$

The mean of $\zeta(k' - k)$ can be computed as:

$$\mathbb{E}[\zeta(k' - k)] = \mathbb{E}\left[\frac{1}{N} \sum_{n=0}^{N-1} e^{j\vartheta[n]} e^{j2\pi(k' - k)n/N}\right] = \frac{1}{N} \sum_{n=0}^{N-1} \mathbb{E}[e^{j\vartheta[n]}] e^{j2\pi(k' - k)n/N} \quad (\text{F.1})$$

$$= \frac{1}{N} e^{-\sigma_{\vartheta}^2/2} \sum_{n=0}^{N-1} e^{j2\pi(k' - k)n/N} \quad (\text{F.2})$$

Here,

$$\frac{1}{N} \sum_{n=0}^{N-1} e^{j2\pi(k' - k)n/N} = \begin{cases} 1 & k' = k \\ \frac{1}{N} \frac{\sin(\pi(k' - k))}{\sin(\pi(k' - k)/N)} e^{j\pi(k' - k)(N-1)/N} & k' \neq k \end{cases} \quad (\text{F.3})$$

Similar to the computation of $\text{var}[\zeta(0)]$, the variance of $\zeta(k' - k)$ can be expressed as:

$$\text{var}[\zeta(k' - k)] = \frac{1}{N^2} \text{var}\left[\sum_{n=0}^{N-1} e^{j\vartheta[n]} e^{j2\pi(k' - k)n/N}\right] = \frac{1}{N^2} \sum_{n=0}^{N-1} \text{var}\left[e^{j\vartheta[n]} e^{j2\pi(k' - k)n/N}\right] \quad (\text{F.4})$$

$$= \frac{1}{N^2} \sum_{n=0}^{N-1} \text{var}\left[e^{j\vartheta[n]}\right] \left|e^{j2\pi(k' - k)n/N}\right|^2 = \frac{1}{N} \left(1 - e^{-\sigma_{\vartheta}^2}\right) \quad (\text{F.5})$$

Proof of *Lemma 3*

Let say ϑ_{TX} and ϑ_{RX} is the phase noise at the transmitter and the receiver, respectively, with same statistics. It is quite obvious that the SDINR expression will be same as in (5.33) when phase noise is present at the receiver only. For the case, when the phase noise is generated at the transmitter only, then the received OFDM signal vector in (5.17) will be:

$$\bar{\mathbf{r}}_u[n] = \sum_{l=0}^{L-1} \bar{\mathbf{H}}_u^l \bar{\mathbf{y}}[n-l] e^{j\vartheta_{TX}[n-l]} + \bar{\mathbf{w}}_u[n]. \quad (\text{G.1})$$

It can easily be seen from (G.1), that only the transmitted signal is affected by the phase noise which leads to the same expressions as derived in (5.26), (5.30) and (5.31). Furthermore, AWGN is not affected by phase noise, which implies that its variance remains same as in (5.32). Thus, the expression of SDINR for this case remains unchanged and similar to (5.33). When the phase noise is present at both transmitter and receiver, then the transmitted signal is multiplied by $e^{j(\vartheta_{TX}[n] + \vartheta_{RX}[n])}$. Since both $\vartheta_{TX}[n]$ and $\vartheta_{RX}[n]$ are $\mathcal{N}(0, \sigma_\vartheta^2)$, thus their sum will be $\mathcal{N}(0, 2\sigma_\vartheta^2)$. However, AWGN is affected by the receiver phase noise only, whose variance remains same as in (5.32). Thus, the expression of SDINR in this case also remains unchanged. Therefore, the proposed analytical framework based on the expression of SDINR given in (5.33) is applicable for any nonlinear MU-MIMO-OFDM system irrespective of the fact whether the phase noise is present at the transmitter or at the receiver or both.

Appendix H

The nonlinear coefficients of (6.7)

$$\delta_1^t = \alpha_1^t \beta_1^t, \quad (\text{H.1})$$

$$\delta_2^t = \alpha_1^t \beta_2^t, \quad (\text{H.2})$$

$$\delta_3^t = \alpha_1^t \beta_3^t + \alpha_3^t \beta_1^t |\beta_1^t|^2, \quad (\text{H.3})$$

$$\delta_4^t = \alpha_1^t \beta_4^t + \alpha_3^t \beta_1^t B_{1,2}^t + \alpha_3^t \beta_2^t |\beta_1^t|^2, \quad (\text{H.4})$$

$$\delta_5^t = \alpha_1^t \beta_5^t + \alpha_3^t \beta_1^t (|\beta_2^t|^2 + B_{1,3}^t) + \alpha_3^t \beta_2^t B_{1,2}^t + \alpha_3^t \beta_3^t |\beta_1^t|^2, \quad (\text{H.5})$$

$$\delta_6^t = \alpha_1^t \beta_6^t + \alpha_3^t \beta_1^t (B_{1,4}^t + B_{2,3}^t) + \alpha_3^t \beta_2^t (|\beta_2^t|^2 + B_{1,3}^t) + \alpha_3^t \beta_3^t B_{1,2}^t + \alpha_3^t \beta_4^t |\beta_1^t|^2, \quad (\text{H.6})$$

$$\begin{aligned} \delta_7^t = & \alpha_3^t \beta_1^t (|\beta_3^t|^2 + B_{1,5}^t + B_{2,4}^t) + \alpha_3^t \beta_2^t (B_{1,4}^t + B_{2,3}^t) + \alpha_3^t \beta_3^t (|\beta_2^t|^2 + B_{1,3}^t) \\ & + \alpha_3^t \beta_4^t B_{1,2}^t + \alpha_3^t \beta_5^t |\beta_1^t|^2, \end{aligned} \quad (\text{H.7})$$

$$\begin{aligned} \delta_8^t = & \alpha_3^t \beta_1^t (B_{1,6}^t + B_{2,5}^t + B_{3,4}^t) + \alpha_3^t \beta_2^t (|\beta_3^t|^2 + B_{1,5}^t + B_{2,4}^t) + \alpha_3^t \beta_3^t (B_{1,4}^t + B_{2,3}^t) \\ & + \alpha_3^t \beta_4^t (|\beta_2^t|^2 + B_{1,3}^t) + \alpha_3^t \beta_5^t B_{1,2}^t + \alpha_3^t \beta_6^t |\beta_1^t|^2, \end{aligned} \quad (\text{H.8})$$

$$\begin{aligned} \delta_9^t = & \alpha_3^t \beta_1^t (|\beta_4^t|^2 + B_{2,6}^t + B_{3,5}^t) + \alpha_3^t \beta_2^t (B_{1,6}^t + B_{2,5}^t + B_{3,4}^t) + \alpha_3^t \beta_3^t (|\beta_3^t|^2 + B_{1,5}^t + B_{2,4}^t) \\ & + \alpha_3^t \beta_4^t (B_{1,4}^t + B_{2,3}^t) + \alpha_3^t \beta_5^t (|\beta_2^t|^2 + B_{1,3}^t) + \alpha_3^t \beta_6^t B_{1,2}^t, \end{aligned} \quad (\text{H.9})$$

$$\begin{aligned} \delta_{10}^t = & \alpha_3^t \beta_1^t (B_{3,6}^t + B_{4,5}^t) + \alpha_3^t \beta_2^t (|\beta_4^t|^2 + B_{2,6}^t + B_{3,5}^t) + \alpha_3^t \beta_3^t (B_{1,6}^t + B_{2,5}^t + B_{3,4}^t) \\ & + \alpha_3^t \beta_4^t (|\beta_3^t|^2 + B_{1,5}^t + B_{2,4}^t) + \alpha_3^t \beta_5^t (B_{1,4}^t + B_{2,3}^t) + \alpha_3^t \beta_6^t (|\beta_2^t|^2 + B_{1,3}^t), \end{aligned} \quad (\text{H.10})$$

$$\begin{aligned} \delta_{11}^t = & \alpha_3^t \beta_1^t (|\beta_5^t|^2 + B_{4,6}^t) + \alpha_3^t \beta_2^t (B_{3,6}^t + B_{4,5}^t) + \alpha_3^t \beta_3^t (|\beta_4^t|^2 + B_{2,6}^t + B_{3,5}^t) \\ & + \alpha_3^t \beta_4^t (B_{1,6}^t + B_{2,5}^t + B_{3,4}^t) + \alpha_3^t \beta_5^t (|\beta_3^t|^2 + B_{1,5}^t + B_{2,4}^t) + \alpha_3^t \beta_6^t (B_{1,4}^t + B_{2,3}^t), \end{aligned} \quad (\text{H.11})$$

$$\begin{aligned}\delta_{12}' = & \alpha_3' \beta_1' B_{5,6}' + \alpha_3' \beta_2' \left(|\beta_5'|^2 + B_{4,6}' \right) + \alpha_3' \beta_3' \left(B_{3,6}' + B_{4,5}' \right) + \alpha_3' \beta_4' \left(|\beta_4'|^2 + B_{2,6}' + B_{3,5}' \right) \\ & + \alpha_3' \beta_5' \left(B_{1,6}' + B_{2,5}' + B_{3,4}' \right) + \alpha_3' \beta_6' \left(|\beta_3'|^2 + B_{1,5}' + B_{2,4}' \right),\end{aligned}\quad (\text{H.12})$$

$$\begin{aligned}\delta_{13}' = & \alpha_3' \beta_1' |\beta_6'|^2 + \alpha_3' \beta_2' B_{5,6}' + \alpha_3' \beta_3' \left(|\beta_5'|^2 + B_{4,6}' \right) + \alpha_3' \beta_4' \left(B_{3,6}' + B_{4,5}' \right) \\ & + \alpha_3' \beta_5' \left(|\beta_4'|^2 + B_{2,6}' + B_{3,5}' \right) + \alpha_3' \beta_6' \left(B_{1,6}' + B_{2,5}' + B_{3,4}' \right),\end{aligned}\quad (\text{H.13})$$

$$\begin{aligned}\delta_{14}' = & \alpha_3' \beta_2' |\beta_6'|^2 + \alpha_3' \beta_3' B_{5,6}' + \alpha_3' \beta_4' \left(|\beta_5'|^2 + B_{4,6}' \right) + \alpha_3' \beta_5' \left(B_{3,6}' + B_{4,5}' \right) \\ & + \alpha_3' \beta_6' \left(|\beta_4'|^2 + B_{2,6}' + B_{3,5}' \right),\end{aligned}\quad (\text{H.14})$$

$$\delta_{15}' = \alpha_3' \beta_3' |\beta_6'|^2 + \alpha_3' \beta_4' B_{5,6}' + \alpha_3' \beta_5' \left(|\beta_5'|^2 + B_{4,6}' \right) + \alpha_3' \beta_6' \left(B_{3,6}' + B_{4,5}' \right), \quad (\text{H.15})$$

$$\delta_{16}' = \alpha_3' \beta_4' |\beta_6'|^2 + \alpha_3' \beta_5' B_{5,6}' + \alpha_3' \beta_6' \left(|\beta_5'|^2 + B_{4,6}' \right), \quad (\text{H.16})$$

$$\delta_{17}' = \alpha_3' \beta_5' |\beta_6'|^2 + \alpha_3' \beta_6' B_{5,6}', \quad (\text{H.17})$$

$$\delta_{18}' = \alpha_3' \beta_6' |\beta_6'|^2, \quad (\text{H.18})$$

where $B_{a,b}' = \beta_a' \beta_b'^* + \beta_a'^* \beta_b'$,

Author's Publications

Journals (Published)

1. **P. Aggarwal** and V. A. Bohara, "End-to-End Theoretical Evaluation of a Nonlinear MIMO-OFDM System in the Presence of Digital Predistorter," IEEE Systems Journal, pp. 1-11, 2018.
2. **P. Aggarwal** and V. A. Bohara, "Analytical Characterization of Dual-Band Multi-User MIMO-OFDM System with Nonlinear Transmitter Constraints," IEEE Transactions on Communications, vol. 66, no. 10, pp. 4536-4549, Oct 2018.
3. **P. Aggarwal** and V. A. Bohara, "On the Multi-Band Carrier Aggregated Nonlinear LTE-A System," IEEE Access, vol. 5, pp. 16930-16943, 2017.
4. **P. Aggarwal** and V. A. Bohara, "A Nonlinear Downlink Multiuser MIMO-OFDM Systems," IEEE Wireless Communications Letters, vol. 6, no. 3, pp. 414-417, June 2017.

Journals (Submitted)

1. **P. Aggarwal**, A. Pradhan, V. A. Bohara, "A Downlink Multi-User MIMO-OFDM System with Non-Ideal Oscillators and Amplifiers: Characterization and Performance Analysis," IEEE Transactions on Wireless Communications, 2018.
2. **P. Aggarwal**, T. Kabra, R. Ahmad, V. A. Bohara, A. Srivastava, "Adaptive Learning Architecture based Predistorter for Nonlinear VLC system," in Photonic Network Communications, 2018.

Conferences (Published)

1. M. H. N. Shaikh, A. Agarwal, **P. Aggarwal**, V. A. Bohara, "On the Spectral Content of Nonlinear Carrier Aggregated Windowed OFDM Systems," 2018 IEEE International Conference on Advanced Networks and Telecommunications Systems (ANTS), Indore, 2018, pp. 1-5.
2. K. Joshi, V. A. Bohara, and **P. Aggarwal**, "Design and Implementation of LTE Advanced Underlay Device-to-Device Communication Framework," in Proceedings of the 24th Annual International Conference on Mobile Computing and Networking (MobiCom '18), ACM, New York, NY, USA, 855-857.
3. **P. Aggarwal**, F. Jabin and V. A. Bohara, "Nonlinear Amplification Effects on Dual Band Multi-User MIMO-OFDM Systems," 2018 IEEE International Conference on Communications (ICC), Kansas City, MO, USA, 2018, pp. 1-6.
4. **P. Aggarwal**, A. Agarwal, V. A. Bohara, "On the Spectral Content of the Nonlinearly Amplified Carrier Aggregated OFDM System," 2018 IEEE Wireless Communications and Networking Conference (WCNC), Barcelona, Spain, 2018, pp. 1-5.
5. **P. Aggarwal**, R. Ahmad, V. A. Bohara, A. Srivastava, "Adaptive Predistortion Technique for nonlinear LED with Dimming Control in VLC system," 2017 IEEE International Conference on Advanced Networks and Telecommunications Systems (ANTS), Bhubaneswar, India, 2017, pp. 1-6.
6. **P. Aggarwal**, A. Agarwal, V. A. Bohara, "Error Vector Magnitude Analysis for Carrier Aggregated OFDM signals with Nonlinear HPA," 2016 IEEE International Conference on Advanced Networks and Telecommunications Systems (ANTS), Bangalore, 2016, pp. 1-3.
7. **P. Aggarwal**, V. A. Bohara, "Characterization of HPA Using Two Dimensional General Memory Polynomial for Dual Band Carrier Aggregated MIMO-OFDM Systems," 2016 IEEE International Conference on Communications (ICC), Kuala Lumpur, 2016, pp. 1-7.
8. P. Singhal, **P. Aggarwal**, V. A. Bohara, "Nonlinear distortion analysis of multi-band carrier aggregated OFDM signals," 2015 IEEE International Conference on Advanced Networks and Telecommunications Systems (ANTS), Kolkata, 2015, pp. 1-6.
9. P. Singhal, **P. Aggarwal**, V. A. Bohara, "Analysis of carrier aggregated OFDM signals in presence of dual band power amplifiers," 2015 Twenty First National Conference on Communications (NCC), Mumbai, 2015, pp. 1-6.

References

- [1] J. Cao, M. Ma, H. Li, Y. Zhang, and Z. Luo, "A survey on security aspects for LTE and LTE-A networks," *IEEE Communications Surveys Tutorials*, vol. 16, no. 1, pp. 283–302, Jan. 2014.
- [2] A. de la Fuente, R. P. Leal, and A. G. Armada, "New technologies and trends for next generation mobile broadcasting services," *IEEE Communications Magazine*, vol. 54, no. 11, pp. 217–223, Nov. 2016.
- [3] Y. Wu and W. Y. Zou, "Orthogonal frequency division multiplexing: A multi-carrier modulation scheme," *IEEE Transactions on Consumer Electronics*, vol. 41, no. 3, pp. 392–399, Aug. 1995.
- [4] Y. Li, J. Winters, and N. Sollenberger, "MIMO-OFDM for wireless communications: Signal detection with enhanced channel estimation," *IEEE Transactions on Communications*, vol. 50, no. 9, pp. 1471–1477, Sep. 2002.
- [5] A. Kiayani, V. Lehtinen, L. Anttila, T. Lahteensuu, and M. Valkama, "Linearity challenges of LTE-Advanced mobile transmitters: Requirements and potential solutions," *IEEE Communications Magazine*, vol. 55, no. 6, pp. 170–179, 2017.
- [6] W. Wang, Z. Zhang, and A. Huang, "Spectrum aggregation: Overview and challenges," *Network Protocols and Algorithms*, vol. 2, no. 1, pp. 184–196, 2010.
- [7] Z. Shen, A. Papasakellariou, J. Montojo, D. Gerstenberger, and F. Xu, "Overview of 3GPP LTE-Advanced carrier aggregation for 4G wireless communications," *IEEE Communications Magazine*, vol. 50, no. 2, pp. 122–130, Feb. 2012.
- [8] E. G. Larsson, O. Edfors, F. Tufvesson, and T. L. Marzetta, "Massive MIMO for next generation wireless systems," *IEEE Communications Magazine*, vol. 52, no. 2, pp. 186–195, Feb. 2014.
- [9] L. Ding, G. Zhou, D. Morgan, Z. Ma, J. Kenney, J. Kim, and C. Giardina, "A robust digital baseband predistorter constructed using memory polynomials," *IEEE Transactions on Communications*, vol. 52, Jan. 2004.
- [10] F. Gregorio, S. Werner, J. Hamalainen, R. Wichman, and J. Cousseau, "BEP analysis of OSTBC-OFDM systems with broadband PA and imperfect memory compensation," *IEEE Communications Letters*, vol. 11, no. 12, pp. 940–942, Dec. 2007.

- [11] D. Dardari, V. Tralli, and A. Vaccari, "A theoretical characterization of nonlinear distortion effects in OFDM systems," *IEEE Transactions on Communications*, vol. 48, no. 10, pp. 1755–1764, Oct. 2000.
- [12] N. Y. Ermolova, "Spectral analysis of nonlinear amplifier based on the complex gain Taylor series expansion," *IEEE Communications Letters*, vol. 5, no. 12, pp. 465–467, Dec. 2001.
- [13] Z.-W. Zheng, Z.-X. Yang, C.-Y. Pan, and Y.-S. Zhu, "Performance analysis of the OFDM-based systems with nonlinear distortion and imperfect parameters estimation," *IEEE Transactions on Consumer Electronics*, vol. 50, no. 3, pp. 835–843, Aug. 2004.
- [14] P. P. Ann and R. Jose, "Comparison of PAPR reduction techniques in OFDM systems," in *2016 International Conference on Communication and Electronics Systems (ICCES)*, Oct. 2016, pp. 1–5.
- [15] S. H. Han and J. H. Lee, "An overview of peak-to-average power ratio reduction techniques for multicarrier transmission," *IEEE Wireless Communications*, vol. 12, no. 2, pp. 56–65, Apr. 2005.
- [16] Y. Rahmatallah and S. Mohan, "Peak-to-average power ratio reduction in OFDM systems: A survey and taxonomy," *IEEE Communications Surveys Tutorials*, vol. 15, no. 4, pp. 1567–1592, Fourth 2013.
- [17] A. Ghosh, R. Ratasuk, B. Mondal, N. Mangalvedhe, and T. Thomas, "LTE-Advanced: Next-generation wireless broadband technology [invited paper]," *IEEE Wireless Communications*, vol. 17, no. 3, 2010.
- [18] H. Lee, S. Vahid, and K. Moessner, "A survey of radio resource management for spectrum aggregation in LTE-Advanced," *IEEE Communications Surveys Tutorials*, vol. 16, no. 2, pp. 745–760, Second 2014.
- [19] S. Rostami, K. Arshad, and P. Rapajic, "Optimum radio resource management in carrier aggregation based LTE-Advanced systems," *IEEE Transactions on Vehicular Technology*, vol. 67, no. 1, pp. 580–589, Jan. 2018.
- [20] P. Aggarwal and V. A. Bohara, "On the multiband carrier aggregated nonlinear LTE-A system," *IEEE Access*, vol. 5, pp. 16 930–16 943, 2017.
- [21] J. Liu and W. Xiao, "Advanced carrier aggregation techniques for multi-carrier ultra-dense networks," *IEEE Communications Magazine*, vol. 54, no. 7, pp. 61–67, Jul. 2016.
- [22] W. Guo, J. Fan, G. Y. Li, Q. Yin, and X. Zhu, "Adaptive SU/MU-MIMO scheduling schemes for LTE-A downlink transmission," *IET Communications*, vol. 11, no. 6, pp. 783–792, 2017.
- [23] S. Akbar, Y. Deng, A. Nallanathan, M. El Kashlan, and G. K. Karagiannidi, "Massive multiuser MIMO in heterogeneous cellular networks with full duplex small cells," *IEEE Transactions on Communications*, vol. 65, no. 11, pp. 4704–4719, Nov. 2017.

- [24] T. L. Marzetta, "Noncooperative cellular wireless with unlimited numbers of base station antennas," *IEEE Transactions on Wireless Communications*, vol. 9, no. 11, pp. 3590–3600, Nov. 2010.
- [25] F. Rusek, D. Persson, B. K. Lau, E. G. Larsson, T. L. Marzetta, O. Edfors, and F. Tufveson, "Scaling up MIMO: Opportunities and challenges with very large arrays," *IEEE Signal Processing Magazine*, vol. 30, no. 1, pp. 40–60, Jan. 2013.
- [26] D. Gesbert, M. Kountouris, R. W. H. Jr., C. b. Chae, and T. Salzer, "Shifting the MIMO Paradigm," *IEEE Signal Processing Magazine*, vol. 24, no. 5, pp. 36–46, Sept 2007.
- [27] L. Lu, G. Y. Li, A. L. Swindlehurst, A. Ashikhmin, and R. Zhang, "An overview of massive MIMO: Benefits and challenges," *IEEE Journal of Selected Topics in Signal Processing*, vol. 8, no. 5, pp. 742–758, Oct. 2014.
- [28] P. Aggarwal and V. A. Bohara, "A nonlinear downlink multiuser MIMO-OFDM systems," *IEEE Wireless Communications Letters*, vol. 6, no. 3, pp. 414–417, Jun. 2017.
- [29] C. B. Peel, B. M. Hochwald, and A. L. Swindlehurst, "A vector-perturbation technique for near-capacity multiantenna multiuser communication-part I: Channel inversion and regularization," *IEEE Transactions on Communications*, vol. 53, no. 1, pp. 195–202, Jan. 2005.
- [30] Q. Spencer, A. Swindlehurst, and M. Haardt, "Zero-forcing methods for downlink spatial multiplexing in multiuser MIMO channels," *IEEE Transactions on Signal Processing*, vol. 52, no. 2, pp. 461–471, Feb. 2004.
- [31] L.-U. Choi and R. D. Murch, "A transmit preprocessing technique for multiuser MIMO systems using a decomposition approach," *IEEE Transactions on Wireless Communications*, vol. 3, no. 1, pp. 20–24, Jan. 2004.
- [32] B. M. Hochwald, C. B. Peel, and A. L. Swindlehurst, "A vector-perturbation technique for near-capacity multiantenna multiuser communication-part II: Perturbation," *IEEE Transactions on Communications*, vol. 53, no. 3, pp. 537–544, Mar. 2005.
- [33] Z. Tu and R. S. Blum, "Multiuser diversity for a dirty paper approach," *IEEE Communications Letters*, vol. 7, no. 8, pp. 370–372, Aug. 2003.
- [34] M. R. Khanzadi, "Phase noise in communication systems," Ph.D. dissertation, Chalmers University of Technology, 2015.
- [35] T. Oh and R. Harjani, "A 6-gb/s mimo crosstalk cancellation scheme for high-speed i/os," *IEEE Journal of Solid-State Circuits*, vol. 46, no. 8, pp. 1843–1856, Aug 2011.
- [36] M. Isaksson, D. Wisell, and D. Ronnow, "A comparative analysis of behavioral models for RF power amplifiers," *IEEE Transactions on Microwave Theory and Techniques*, vol. 54, no. 1, pp. 348–359, Jan. 2006.
- [37] F. M. Ghannouchi and O. Hammi, "Behavioral modeling and predistortion," *IEEE Microwave Magazine*, vol. 10, no. 7, pp. 52–64, Dec. 2009.

- [38] J. Kim and K. Konstantinou, "Digital predistortion of wideband signals based on power amplifier model with memory," *Electronics Letters*, vol. 37, no. 23, pp. 1417–1418, Nov. 2001.
- [39] C. Rapp, "Effects of HPA-Nonlinearity on a 4-DPSK/OFDM-Signal for a digital sound broadcasting system," *Proceedings Second European Conf. on Sat. Comm. (ESA SP-332)*, pp. 179–184, 1991.
- [40] A. A. M. Saleh, "Frequency-independent and frequency-dependent nonlinear models of TWT amplifiers," *IEEE Transactions on Communications*, vol. 29, no. 11, pp. 1715–1720, Nov. 1981.
- [41] V. Bohara and S. H. Ting, "Theoretical analysis of OFDM signals in nonlinear polynomial models," in *2007 6th International Conference on Information, Communications Signal Processing*, Dec. 2007, pp. 1–5.
- [42] H. Ku and J. Kenney, "Behavioral modeling of nonlinear RF power amplifiers considering memory effects," *IEEE Transactions on Microwave Theory and Techniques*, vol. 51, no. 12, pp. 2495–2504, Dec. 2003.
- [43] A. S. Tehrani, H. Cao, S. Afsardoost, T. Eriksson, M. Isaksson, and C. Fager, "A comparative analysis of the complexity/accuracy tradeoff in power amplifier behavioral models," *IEEE Transactions on Microwave Theory and Techniques*, vol. 58, no. 6, pp. 1510–1520, Jun. 2010.
- [44] D. Morgan, Z. Ma, J. Kim, M. Zierdt, and J. Pastalan, "A generalized memory polynomial model for digital predistortion of RF power amplifiers," *IEEE Transactions on Signal Processing*, vol. 54, pp. 3852–3860, Oct. 2006.
- [45] A. E. Nordsjo and L. H. Zetterberg, "Identification of certain time-varying nonlinear Wiener and Hammerstein systems," *IEEE Transactions on Signal Processing*, vol. 49, no. 3, pp. 577–592, Mar. 2001.
- [46] A. Demir, A. Mehrotra, and J. Roychowdhury, "Phase noise in oscillators: A unifying theory and numerical methods for characterization," *IEEE Transactions on Circuits and Systems I: Fundamental Theory and Applications*, vol. 47, no. 5, pp. 655–674, May 2000.
- [47] A. G. Armada, "Understanding the effects of phase noise in orthogonal frequency division multiplexing (OFDM)," *IEEE Transactions on Broadcasting*, vol. 47, no. 2, pp. 153–159, Jun. 2001.
- [48] A. K. Kwan, M. Younes, R. Darraji, and F. M. Ghannouchi, "On track for efficiency: Concurrent multiband envelope-tracking power amplifiers," *IEEE Microwave Magazine*, vol. 17, no. 5, pp. 46–59, May 2016.
- [49] B. Kim, J. Kim, D. Kim, J. Son, Y. Cho, J. Kim, and B. Park, "Push the envelope: Design concepts for envelope-tracking power amplifiers," *IEEE Microwave Magazine*, vol. 14, no. 3, pp. 68–81, May 2013.
- [50] Z. Wang, "Demystifying envelope tracking: Use for high-efficiency power amplifiers for 4G and beyond," *IEEE Microwave Magazine*, vol. 16, no. 3, pp. 106–129, Apr. 2015.

- [51] R. N. Braithwaite, "Analog linearization techniques suitable for rf power amplifiers used in integrated transmitters," in *2013 IEEE Compound Semiconductor Integrated Circuit Symposium (CSICS)*, Oct 2013, pp. 1–4.
- [52] S. Rezaei, M. S. Hashmi, B. Dehlaghi, and F. M. Ghannouchi, "A systematic methodology to design analog predistortion linearizer for dual inflection power amplifiers," in *2011 IEEE MTT-S International Microwave Symposium*, June 2011, pp. 1–4.
- [53] M. S. Hashmi, A. L. Clarke, J. Lees, M. Helaoui, P. J. Tasker, and F. M. Ghannouchi, "Agile harmonic envelope load-pull system enabling reliable and rapid device characterization," *Measurement Science and Technology*, vol. 21, no. 5, p. 055109, Mar 2010. [Online]. Available: <https://doi.org/10.1088%2F0957-0233%2F21%2F5%2F055109>
- [54] V. A. Bohara, M. A. Hussein, and O. Venard, "A parameter identification algorithm for multi-stage digital predistorter," in *2013 European Microwave Conference (EuMC)*, Oct. 2013, pp. 416–419.
- [55] M. A. Hussein, V. A. Bohara, and O. Venard, "Multi-stage digital predistortion based on indirect learning architecture," in *2013 IEEE International Conference on Acoustics, Speech and Signal Processing (ICASSP)*, May 2013, pp. 6093–6097.
- [56] —, "On the system level convergence of ILA and DLA for digital predistortion," in *2012 International Symposium on Wireless Communication Systems (ISWCS)*, Aug. 2012, pp. 870–874.
- [57] —, "Two-dimensional memory selective polynomial model for digital predistortion," in *2012 IEEE 10th International New Circuits and Systems Conference (NEWCAS)*, Jun. 2012, pp. 469–472.
- [58] S. A. Bassam, M. Helaoui, and F. M. Ghannouchi, "Crossover digital predistorter for the compensation of crosstalk and nonlinearity in mimo transmitters," *IEEE Transactions on Microwave Theory and Techniques*, vol. 57, no. 5, pp. 1119–1128, May 2009.
- [59] S. Amin, P. N. Landin, P. Händel, and D. Rönnow, "Behavioral modeling and linearization of crosstalk and memory effects in rf mimo transmitters," *IEEE Transactions on Microwave Theory and Techniques*, vol. 62, no. 4, pp. 810–823, Apr. 2014.
- [60] A. Abdelhafiz, L. Behjat, F. M. Ghannouchi, M. Helaoui, and O. Hammi, "A high-performance complexity reduced behavioral model and digital predistorter for mimo systems with crosstalk," *IEEE Transactions on Communications*, vol. 64, no. 5, pp. 1996–2004, May 2016.
- [61] A. Vaezi, A. Abdipour, A. Mohammadi, and F. M. Ghannouchi, "On the Modeling and Compensation of Backward Crosstalk in MIMO Transmitters," *IEEE Microwave and Wireless Components Letters*, vol. 27, no. 9, pp. 842–844, Sept 2017.
- [62] P. B. Kenington, "Linearized transmitters: an enabling technology for software defined radio," *IEEE Communications Magazine*, vol. 40, no. 2, pp. 156–162, Feb. 2002.
- [63] H. Ochiai and H. Imai, "Performance analysis of deliberately clipped OFDM signals," *IEEE Transactions on Communications*, vol. 50, no. 1, pp. 89–101, Jan. 2002.

- [64] P. Banelli, "Theoretical analysis and performance of OFDM signals in nonlinear fading channels," *IEEE Transactions on Wireless Communications*, vol. 2, no. 2, pp. 284–293, Mar. 2003.
- [65] V. Bohara and S. H. Ting, "Analysis of OFDM signals in nonlinear high power amplifier with memory," in *2008 IEEE International Conference on Communications (ICC)*, May 2008, pp. 3653–3657.
- [66] —, "Analytical performance of orthogonal frequency division multiplexing systems impaired by a non-linear high-power amplifier with memory," *IET Communications*, vol. 3, no. 10, pp. 1659–1666, Oct. 2009.
- [67] J. Qi and S. Aissa, "On the power amplifier nonlinearity in MIMO transmit beamforming systems," *IEEE Transactions on Communications*, vol. 60, no. 3, pp. 876–887, Mar. 2012.
- [68] H. Hemesi, A. Abdipour, and A. Mohammadi, "Analytical modeling of MIMO-OFDM system in the presence of nonlinear power amplifier with memory," *IEEE Transactions on Communications*, vol. 61, no. 1, pp. 155–163, Jan. 2013.
- [69] T. C. W. Schenk, P. F. M. Smulders, and E. R. Fledderus, "Impact of nonlinearities in multiple-antenna OFDM transceivers," in *2006 Symposium on Communications and Vehicular Technology*, Nov. 2006, pp. 53–56.
- [70] I. Iofedov and D. Wulich, "MIMO-OFDM with nonlinear power amplifiers," *IEEE Transactions on Communications*, vol. 63, no. 12, pp. 4894–4904, Dec. 2015.
- [71] A. K. Ajami, H. A. Artail, and M. M. Mansour, "PAPR reduction in LTE-Advanced carrier aggregation using low-complexity joint interleaving technique," in *2015 IEEE Wireless Communications and Networking Conference (WCNC)*, Mar. 2015, pp. 675–680.
- [72] P. Yen, H. Minn, and C. C. Chong, "PAPR reduction for bandwidth-aggregated OFDM and SC-FDMA systems," in *2011 IEEE Wireless Communications and Networking Conference (WCNC)*, Mar. 2011, pp. 1363–1368.
- [73] P. Yen and H. Minn, "Low complexity PAPR reduction methods for carrier-aggregated MIMO OFDMA and SC-FDMA systems," *EURASIP Journal on Wireless Communications and Networking*, vol. 2012, no. 1, pp. 1–13, 2012.
- [74] 3GPP R1-094607, "CM/PAPR reduction of aggregated carriers for uplink of LTE-Advanced," *Alcatel-Lucent*, Nov. 2009.
- [75] A. k. Ajami, H. A. Artail, and M. M. Mansour, "A low-complexity PAPR reduction technique for LTE-Advanced uplink with carrier aggregation," in *2015 IEEE Global Communications Conference (GLOBECOM)*, Dec. 2015, pp. 1–6.
- [76] J. Moon and B. Kim, "Enhanced Hammerstein behavioral model for broadband wireless transmitters," *IEEE Transactions on Microwave Theory and Techniques*, vol. 59, no. 4, pp. 924–933, 2011.

- [77] S. Bassam, M. Helaoui, and F. Ghannouchi, "2-D Digital predistortion (2-D-DPD) architecture for concurrent dual-band transmitters," *IEEE Transactions on Microwave Theory and Techniques*, vol. 59, pp. 2547–2553, Oct. 2011.
- [78] A. Vaezi, A. Abdipour, A. Mohammadi, and F. M. Ghannouchi, "Analysis of MIMO-OFDM system impaired by nonlinear dual-band power amplifiers," *Analog Integrated Circuits and Signal Processing*, vol. 89, no. 1, pp. 205–212, 2016. [Online]. Available: <http://dx.doi.org/10.1007/s10470-016-0821-2>
- [79] M. Jiang and L. Hanzo, "Multiuser MIMO-OFDM for next-generation wireless systems," *Proceedings of the IEEE*, vol. 95, no. 7, pp. 1430–1469, Jul. 2007.
- [80] S. Schwarz and M. Rupp, "Evaluation of distributed multi-user MIMO-OFDM With limited feedback," *IEEE Transactions on Wireless Communications*, vol. 13, no. 11, pp. 6081–6094, Nov. 2014.
- [81] P. Xu, J. Wang, J. Wang, and F. Qi, "Analysis and design of channel estimation in multicell multiuser MIMO OFDM systems," *IEEE Transactions on Vehicular Technology*, vol. 64, no. 2, pp. 610–620, Feb. 2015.
- [82] Y. Jang, J. Joung, W. Shin, and E. Jeong, "Frame design and throughput evaluation for practical multiuser MIMO OFDMA systems," *IEEE Transactions on Vehicular Technology*, vol. 60, no. 7, pp. 3127–3141, Sept 2011.
- [83] S. Parkvall, E. Dahlman, A. Furuskar, and M. Frenne, "NR: The new 5G radio access technology," *IEEE Communications Standards Magazine*, vol. 1, no. 4, pp. 24–30, Dec. 2017.
- [84] A. F. Cattoni, H. T. Nguyen, J. Duplicy, D. Tandur, B. Badic, R. Balraj, F. Kaltenberger, I. Latif, A. Bhamri, G. Vivier *et al.*, "Multi-user MIMO and carrier aggregation in 4G systems: The SAMURAI approach," in *2012 IEEE Wireless Communications and Networking Conference Workshops (WCNCW)*, 2012, pp. 288–293.
- [85] S. Umeda, S. Suyama, H. Suzuki, and K. Fukawa, "PAPR reduction method for block diagonalization in multiuser MIMO-OFDM systems," in *2010 IEEE 71st Vehicular Technology Conference (VTC 2010-Spring)*, May 2010, pp. 1–5.
- [86] C. Ni, Y. Ma, and T. Jiang, "A novel adaptive tone reservation scheme for PAPR reduction in large-scale multi-user MIMO-OFDM systems," *IEEE Wireless Communications Letters*, vol. 5, no. 5, pp. 480–483, Oct. 2016.
- [87] C. Studer and E. G. Larsson, "PAR-Aware large-scale multi-user MIMO-OFDM downlink," *IEEE Journal on Selected Areas in Communications*, vol. 31, no. 2, pp. 303–313, Feb. 2013.
- [88] S. Wu and Y. Bar-Ness, "OFDM systems in the presence of phase noise: Consequences and solutions," *IEEE Transactions on Communications*, vol. 52, no. 11, pp. 1988–1996, Nov. 2004.
- [89] H.-G. Ryu, Y. S. Li, and J.-S. Park, "Nonlinear analysis of the phase noise in the OFDM communication system," *IEEE Transactions on Consumer Electronics*, vol. 50, no. 1, pp. 54–63, Feb. 2004.

- [90] D. Petrovic, W. Rave, and G. Fettweis, "Effects of phase noise on OFDM systems with and without PLL: Characterization and compensation," *IEEE Transactions on Communications*, vol. 55, no. 8, pp. 1607–1616, Aug. 2007.
- [91] H. A. Suraweera, X. Zhou, and J. Armstrong, "On the calculation of OFDM error performance with phase noise in AWGN and fading channels," in *2006 IEEE 63rd Vehicular Technology Conference*, vol. 3, May 2006, pp. 1421–1425.
- [92] A. Pitarokoilis, S. K. Mohammed, and E. G. Larsson, "Effect of oscillator phase noise on uplink performance of large MU-MIMO systems," in *2012 50th Annual Allerton Conference on Communication, Control, and Computing (Allerton)*, Oct. 2012, pp. 1190–1197.
- [93] R. Krishnan, M. R. Khanzadi, N. Krishnan, Y. Wu, A. G. i Amat, T. Eriksson, and R. Schober, "Linear massive MIMO precoders in the presence of phase noise-A large-scale analysis," *IEEE Transactions on Vehicular Technology*, vol. 65, no. 5, pp. 3057–3071, May 2016.
- [94] R. Corvaja and A. G. Armada, "Phase Noise Degradation in Massive MIMO Downlink With Zero-Forcing and Maximum Ratio Transmission Precoding," *IEEE Transactions on Vehicular Technology*, vol. 65, no. 10, pp. 8052–8059, Oct. 2016.
- [95] R. Krishnan, M. R. Khanzadi, N. Krishnan, A. G. i Amat, T. Eriksson, N. Mazzali, and G. Colavolpe, "On the impact of oscillator phase noise on the uplink performance in a massive MIMO-OFDM system," *CoRR*, vol. abs/1405.0669, 2014. [Online]. Available: <http://arxiv.org/abs/1405.0669>
- [96] E. Costa and S. Pupolin, "M-QAM-OFDM system performance in the presence of a nonlinear amplifier and phase noise," *IEEE Transactions on Communications*, vol. 50, no. 3, pp. 462–472, Mar. 2002.
- [97] A. Chorti and M. Brookes, "On the effects of memoryless nonlinearities on M-QAM and DQPSK OFDM signals," *IEEE Transactions on Microwave Theory and Techniques*, vol. 54, no. 8, pp. 3301–3315, Aug. 2006.
- [98] M. Madani, A. Abdipour, and A. Mohammadi, "Analysis of performance degradation due to non-linearity and phase noise in orthogonal frequency division multiplexing systems," *IET Communications*, vol. 4, no. 10, pp. 1226–1237, Jul. 2010.
- [99] M. H. Madani, A. Abdipour, and A. Mohammadi, "Analysis of effect of high-power amplifier modelling on the performance of orthogonal frequency division multiplexing transceiver for wireless communications," *IET Science, Measurement Technology*, vol. 6, no. 3, pp. 159–167, May 2012.
- [100] S. Bittner, P. Zillmann, and G. Fettweis, "Equalisation of MIMO-OFDM signals affected by phase noise and clipping and filtering," in *2008 IEEE International Conference on Communications*, May 2008, pp. 609–614.
- [101] A. Papoulis, "Probability, random variables, and stochastic processes," New York, 1965.

- [102] N. Y. Ermolova and O. Tirkkonen, "Theoretical characterization of memory polynomial models with gaussian inputs," *IEEE Signal Processing Letters*, vol. 16, no. 8, pp. 651–654, Aug 2009.
- [103] D. Gore, R. Heath, and A. Paulraj, "Transmit selection in spatial multiplexing systems," *IEEE Communications Letters*, vol. 6, no. 11, pp. 491–493, Nov. 2002.
- [104] J. Proakis and M. Salehi, *Digital Communications*, 5th ed. McGraw-Hill, 2008.
- [105] I. S. Gradshteyn and I. M. Ryzhik, *Table of integrals, series, and products*, 7th ed. Elsevier/Academic Press, Amsterdam, 2007.
- [106] E. W. Ng and M. Geller, "A table of integrals of the error functions," *Journal of Research of the National Bureau of Standards*, pp. 1–20, 1969.
- [107] M. S. Heutmaker, "The error vector and power amplifier distortion," in *1997 Proceedings in Wireless Communications Conference (WCC)*, Aug. 1997, pp. 100–104.
- [108] K. M. Gharaibeh, K. G. Gard, and M. B. Steer, "Accurate estimation of digital communication system metrics - SNR, EVM and ρ ; in a nonlinear amplifier environment," in *64th ARFTGF Microwave Measurements Conference*, Dec. 2004, pp. 41–44.
- [109] R. A. Shafik, M. S. Rahman, and A. R. Islam, "On the extended relationships among EVM, BER and SNR as performance metrics," in *2006 International Conference on Electrical and Computer Engineering*, Dec. 2006, pp. 408–411.
- [110] Z. Qijun, X. Qinghua, and Z. Wei, "A new EVM calculation method for broadband modulated signals and simulation," in *8th International Conference on Electronic Measurement and Instruments (ICEMI)*, Aug. 2007, pp. 661–665.
- [111] 3GPP TS 36.101 V12.5.0, "UE radio transmission and reception."
- [112] 3GPP TS 36.104 V11.2.0, "Base station radio transmission and reception."
- [113] P. Aggarwal and V. A. Bohara, "Characterization of HPA using two dimensional general memory polynomial for dual band carrier aggregated MIMO-OFDM systems," in *2016 IEEE International Conference on Communications (ICC)*, May 2016, pp. 1–7.
- [114] G. Strang, *Linear algebra and its applications*, 4th ed. Thomson, Brooks/Cole, 2006.
- [115] E. K. S. Au, S. Jin, M. R. McKay, W. H. Mow, X. Gao, and I. B. Collings, "Analytical performance of MIMO-SVD systems in Ricean fading channels with channel estimation error and feedback delay," *IEEE Transactions on Wireless Communications*, vol. 7, no. 4, pp. 1315–1325, Apr. 2008.
- [116] E. Björnson, J. Hoydis, M. Kountouris, and M. Debbah, "Massive MIMO systems with non-ideal hardware: Energy efficiency, estimation, and capacity limits," *IEEE Transactions on Information Theory*, vol. 60, no. 11, pp. 7112–7139, Nov. 2014.
- [117] J. Hoydis, S. ten Brink, and M. Debbah, "Massive MIMO in the UL/DL of cellular networks: How many antennas do we need?" *IEEE Journal on Selected Areas in Communications*, vol. 31, no. 2, pp. 160–171, Feb. 2013.

- [118] T. L. Narasimhan, P. Raviteja, and A. Chockalingam, "Large-scale multiuser SM-MIMO versus massive MIMO," in *2014 Information Theory and Applications Workshop (ITA)*, Feb. 2014, pp. 1–9.
- [119] H. Shin and J. H. Lee, "Capacity of multiple-antenna fading channels: Spatial fading correlation, double scattering, and keyhole," *IEEE Transactions on Information Theory*, vol. 49, no. 10, pp. 2636–2647, Oct. 2003.
- [120] P. Aggarwal and V. A. Bohara, "Analytical characterization of dual-band multi-user MIMO-OFDM system with nonlinear transmitter constraints," *IEEE Transactions on Communications*, vol. 66, no. 10, pp. 4536–4549, Oct. 2018.
- [121] H. E. Rowe, "Memoryless nonlinearities with Gaussian inputs: Elementary results," *The Bell System Technical Journal*, vol. 61, no. 7, pp. 1519–1525, Sept 1982.
- [122] E. Balti and M. Guizani, "Impact of non-linear high-power amplifiers on cooperative relaying systems," *IEEE Transactions on Communications*, vol. 65, no. 10, pp. 4163–4175, Oct. 2017.
- [123] H. A. R. Barnett, "The variance of the product of two independent variables and its application to an investigation based on sample data," *Journal of the Institute of Actuaries*, vol. 81, no. 2, p. 190–190, 1955.
- [124] P. Aggarwal, F. Jabin, and V. A. Bohara, "Nonlinear amplification effects on dual band multi-user MIMO-OFDM systems," in *2018 IEEE International Conference on Communications (ICC)*, May 2018, pp. 1–6.
- [125] Y. H. Lim, Y. S. Cho, I. W. Cha, and D. H. Youn, "An adaptive nonlinear prefilter for compensation of distortion in nonlinear systems," *IEEE Transactions on Signal Processing*, vol. 46, no. 6, pp. 1726–1730, Jun. 1998.
- [126] E. Bogenfeld, R. Valentin, K. Metzner, and W. Sauer-Greff, "Influence of nonlinear HPA on trellis-coded OFDM for terrestrial broadcasting of digital HDTV," in *1993 IEEE Global Communications Conference (GLOBECOM)*, Nov. 1993, pp. 1433–1438 vol.3.
- [127] N. Kelly and A. Zhu, "Low-complexity stochastic optimization-based model extraction for digital predistortion of RF power amplifiers," *IEEE Transactions on Microwave Theory and Techniques*, vol. 64, no. 5, pp. 1373–1382, May 2016.