Minimizing Age in IoT Networks With Gateway Based Aggregation

By

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Certificate

This is to certify that the thesis titled **Minimizing Age in IoT Networks With Gateway Based Aggregation** being submitted by **K Aditi** to the Indraprastha Institute of Information Technology Delhi, for the award of the Master of Technology, is an original research work carried out by her under my supervision. In my opinion, the thesis has reached the standards fulfilling the requirements of the regulations relating to the degree.

The results contained in this thesis have not been submitted in part or full to any other university or institute for the award of any degree/diploma.

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ABSTRACT

Real-time monitoring applications have networks of Internet-of-Things (IoT) devices sense and communicate information from a scene of interest to a monitor, for example, a server in the cloud. Often the sensed information is first communicated by the sensors, over an access network, to an IoT gateway that aggregates and sends the sensed information to the monitor. The monitor would like to maximize the freshness of sensed information at its end. In this work, we use the metric of age of information to quantify freshness at the monitor.

Earlier work has considered a set of polling policies, named Poll-s, that have the gateway poll one sensor for fresh information at a time and have the gateway send the polled information to the monitor after a certain a priori fixed number s of sensors have been polled. In our work, we (a) analyze a simpler randomized policy that unlike Poll-s doesn't require knowledge of the vector of ages of information at the monitor at every decision instant, (b) derive a lower bound on the achievable average age at the monitor, (c) propose a heuristic policy that at every decision instant chooses to poll a sensor or transmit to the monitor such that the average drift in age at the monitor is minimized, and (d) propose improvements via optimal ordering of polling of sensors over the best earlier proposed Poll-s policies for when the random times taken to poll sensors are not identically distributed. We show empirically over a wide range of assumed distributions of polling times of sensors, and the time taken to transmit to the monitor, that the randomized policy achieves an age that is within 2.5× the lower bound. We also provide a detailed comparison of the different policies.

Contents

Chapter 1

Introduction

Networks of IoT devices generate time sensitive information updates, which are sent over the internet to monitors (cloud or edge servers) for applications like real-time monitoring, actuation, and analytics. The updates may contain, for example, locations of individuals being tracked, locations and speeds of vehicles, and the temperature field of a physical environment being monitored. The updates are processed for further control and decision making, for example, safe on-road navigation and ambient temperature control.

These updates are different from typical file data, voice and video, in that for the aforementioned applications it is paramount that information at the monitor is 'fresh'. Also, such applications are resilient to lost updates. A new update is more desirable than a retransmitted or backlogged old update. For example, if an update packet containing temperature information is backlogged in the network, and by the time it reaches its destination a new update containing more up to date temperature information is received by the monitor, the older packet's information when received is not fresh and maybe discarded, given the purpose of real-time monitoring.

In this work, we use the metric of age of information (AoI) to measure timeliness. It measures the freshness of data from the perspective of the monitor (destination of updates). At any given time, the age of updates at the monitor is the time elapsed since the generation time of the freshest update available at the monitor. Age at the monitor increases linearly in absence of reception of fresher updates.

We will consider sensors that communicate their information with the monitor via a gateway. All sensors can communicate with the gateway, which then aggregates the sensed information before sending it to the monitor as one transmission. Our work looks at a constrained setting wherein either exactly one sensor can communicate with the gateway or the gateway can send to the monitor. This could, for example, model a scenario where the gateway has a single wireless interface communicating with the sensors and the gateway.

Note that such a setting was first looked at in [1]. An interesting takeaway from [1] was that the sensor polling times and update transmission times to the monitor impact how often the gateway must aggregate sensed information and transmit to the monitor. As shown in [1], the extremes of sending to the monitor after every polling and sending to the monitor after all sensors have been polled are in general detrimental to minimizing age in the gateway based setting. The authors proposed Poll-s policies that have the gateway poll s sensors for their updates before transmitting the polled updates to the monitor. The Poll-s policy is discussed in detail in Chapter 5. In this work, we make the following contributions to understanding timeliness in such network settings.

1. We analyze the randomized policy, which has the gateway either choose to poll a sensor or transmit to the monitor with a priori fixed probabilities. The policy is simpler than Poll-s in that it doesn't require the knowledge of age of sensors' updates at the monitor and the gateway for decision making.

2. We propose a policy that instead minimizes the drift for a linear Lyapunov function. This policy requires knowledge of ages of sensors' updates at the monitor and the gateway at decision instants.

3. For when the number n of sensors is an integral multiple of s, where s is the number of sensors are polled before sending to gateway, we derive the optimal order in which sensors that have non-identical polling time distributions must be polled. The ordering of polling is shown to lead to smaller average age in comparison to the best heuristic Poll-s based policy proposed in [1].

4. We also derive an expression for the lower bound on the age of information that may be achieved by any causal policy. We compare the bound with the other policies proposed in this thesis and in [1] and empirically show that the

worst performing randomized policy leads to AoI that is within 2.5 times the lower bound.

Chapter 2

Related Works

One of the first works to analyze Age of Information as a timeliness metric was [2]. Considering the queuing discipline of *first-come-first-served* (FCFS), their work showed the existence of an optimal rate at which a source must generate its information so that the information remains as fresh as possible up on reception at the destination. Since then AoI has received a lot of attention. Various recent works have investigated the problem of scheduling of nodes in wireless networks. In [3], the authors attempt to find policies that minimize AoI in wireless broadcast networks with unreliable channels. In [4], the authors develop policies that minimize AoI subject to minimum throughput requirements. In [5], the authors study link scheduling using AoI minimization in unreliable networks with stochastic packet arrivals which operate under different queuing disciplines. Low-complexity policies like Randomized policy, Age-based Max Weight policy and Whittle's Index policy were developed in [3–5]. The latter two policies were shown to perform close to optimal. The Whittle's Index policy was also investigated in [6], [7].

The network considered in all of these works is a single-hop network. The authors in [1] consider a network of status updating sensors whose updates are first collected by a gateway and then sent to a monitor. The presence of a gateway makes their network a two-hop network as opposed to a single-hop network in [3–5]. Assuming the gateway to be fixed, they investigate scheduling policies that minimize AoI, one for when sensor polling times are iid and another when sensor polling times may be non-identically distributed. Motivated by the inclusion of gateway in the network model, we consider their model to further develop some more scheduling policies at the gateway considering different distributions of sensor polling times. A study on general multi-source, multi-hop wireless network is done in [8] but only for the case where the packet transmission times are fixed.

There has been a lot of research on using AoI for networks where multiple sources contend for shared channel in order to transmit their freshly generated updates. Decentralized update policies, where a node can transmit without having to communicate with BS or any other nodes in the network, have been analyzed for large IoT networks in [9–11]. In [11], a threshold-based age-dependent random access is proposed. While this policy shows improvement over age-independent random access, the analysis is only limited to symmetric network. Random access for queue based network is studied from the point of AoI in [12]. AoI for ALOHA-like random access was studied in [13] and was compared with scheduled access. A version of slotted-ALOHA known as Irregular Repetition Slotted ALOHA showed improved AoI performance as reported in [14]. When packet arrival rate of M transmitters increases beyond $\frac{1}{eM}$, age-based threshold policy with CSMA is shown to perform as well as Max-weight policy which is a centralized scheduling policy and therefore, collision free [15]. Multi-access using CSMA alone is studied in [16] where closed form expression for average age is found using a stochastic hybrid system model. The transmission time of packets was limited to exponential distribution. The authors conclude that even with optimal back-off times, CSMA does not display AoI performance comparable to other multi-access schemes for some special cases.

Chapter 3

Optimization Problem

3.1 System Model

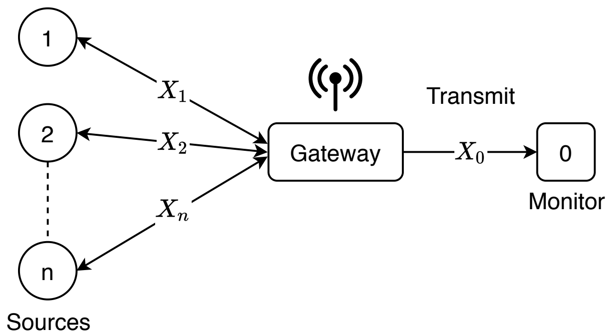


Figure 3.1: We have n sensors, a gateway, and a monitor. Sensor i's polling time is distributed as $f_{X_i}(x)$. Transmission to the monitor is distributed as $f_{X_0}(x)$.

We have a network with n sensors generating time sensitive information, a gateway which could, for example, be a wireless access point, and a monitor as shown in Figure 3.1. The sensors are indexed $1, 2, 3, \ldots, n$ and the monitor is indexed 0. The gateway can either decide to poll a sensor for an update or transmit updates it has already obtained, via polling one or more sensors earlier, to the monitor. The gateway makes a decision at the end of the current poll or transmission to the monitor.

In practice, the act of polling requires the gateway to send a message to the sensor and for the sensor to revert with transmitting an update. We consider a simplified abstraction in which polling a sensor i takes a random X_i amount of time distributed as f_{X_i}. We assume that on being polled the sensor sends back a fresh update and the update is aged by the time X_i on reception by the gateway. The gateway takes time X_0 distributed as f_{X_0} to transmit aggregated updates to the monitor. Note that the random times may capture amongst other things different update lengths and retransmissions by layer 2 of the OSI stack.

In this work, we will say that the sensors are *homogeneous* in case the polling times of all sensors are identically distributed. Otherwise, we will say that the sensors are *heterogeneous*. For either case, the time to transmission to the monitor may be non-identical to polling times. Also, all polling times and the transmission time to the monitor are assumed to be mutually independent. We assume that the distributions are time-invariant. Last but not the least, in this work we restrict ourselves to a static deployment of sensors, gateway, and monitor.

3.2 Problem Formulation

Let $\Delta_i(t)$ be the age of updates of sensor i at the monitor. If $u_i(t)$ is the timestamp (time of generation at sensor i) of the most recent update of i at the monitor, then $\Delta_i(t) = t - u_i(t)$. The function $u_i(t)$ is a step function whose value remains constant until a fresher update from sensor i is received at the monitor. Correspondingly, let $\Delta_i^{'}(t)$ be the age of status updates from sensor i at the gateway. We assume that $\Delta_i(0) = \Delta_i^{'}(0)$ for all sensors i. Let $b_k, k \geq 1$ denote the time instants when the monitor receives updates from the gateway. At such times the age of all sensors at the monitor gets reset to the age of the sensors at the gateway. That is

$$\Delta_i(b_k) = \Delta_i^{'}(b_k) \tag{3.1}$$

Suppose sensor j is polled by the gateway at time τ_j and this results in it receiving the update at $\tau_j^{'}$ after a random X_j amount of time. At the end of the polling, the ages of the sensors' updates at the gateway are given by

$$\Delta_i^{'}(\tau_j^{'}) = \begin{cases} X_j & i = j \\ \Delta_i^{'}(\tau_j) + X_j & i \neq j \end{cases} \tag{3.2}$$

Transmission to the monitor increases all sensors' ages at the gateway by X_0. In the absence of new updates at the gateway, $\Delta_i^{'}(t)$ increases linearly with rate 1. Similarly $\Delta_i(t)$ increases at the gateway in the absence of updates. Note that sensors

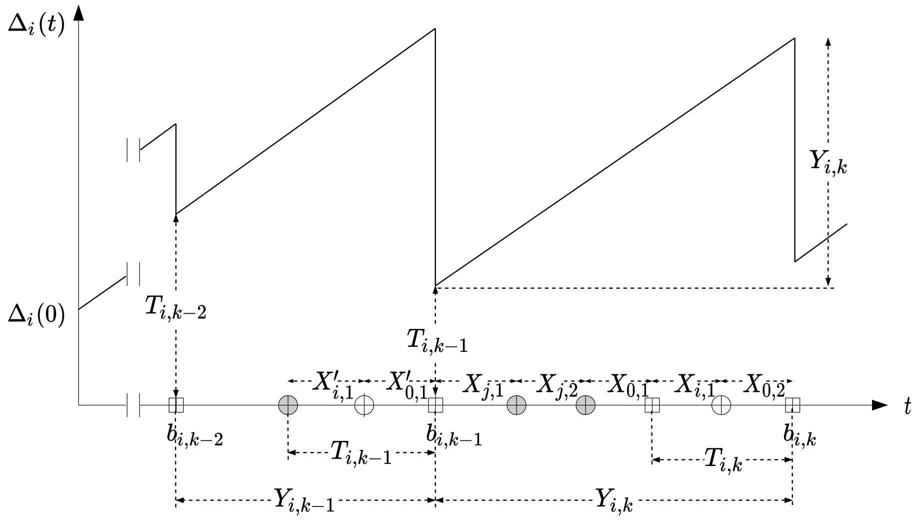


Figure 3.2: Sample function of age of sensor i at the monitor. The empty circles are time instants when a fresh update from i was received by the gateway. The filled circles correspond to the same for another sensor j. The empty squares are when the monitor receives a transmission from the gateway.

that do not get polled by the gateway in (b_{k-1}, b_k) will experience no reduction in age at the monitor at b_k.

The time average age $\bar{\Delta}_i$ of sensor i at the monitor is

$$\bar{\Delta}_i = \lim_{t\to\infty} \frac{1}{t} \int\limits_0^t \Delta_i(t) \tag{3.3}$$

Let $b_{i,k},\ k \geq 1$ denote the time instant when monitor receives k^{th} such transmission from gateway that leads to reduction of age of sensor i at monitor. Let $Y_{i,k}$ denote the time between $k-1^{th}$ and k^{th} reset of age of sensor i at monitor. Let $T_{i,k}$ be the age of the update from sensor i when it is received by the monitor at $b_{i,k}$. The age $\Delta_i(t)$ resets to $T_{i,k}$ at $t = b_{i,k}$. Both $Y_{i,k}$ and $T_{i,k}$ have been marked in Figure 3.2. From [1], we can rewrite the average age $\bar{\Delta}_i$, given by (3.3), as

$$\bar{\Delta}_i = \frac{1}{E[Y_i]} \left[E[Y_i T_i] + \frac{E[Y_i^2]}{2} \right] \tag{3.4}$$

We will consider a set of causal policies that do not use knowledge of the future for decision making. Denote this set of policies by Π and any policy within this set as π. The age of information (AoI) of the system at the monitor on using policy π is

$$\bar{\Delta}^\pi = \frac{1}{n} E\left[\sum_{i=1}^{n} \bar{\Delta}_i \right] \tag{3.5}$$

We would like to find the policy $\pi^* \in \Pi$ that minimizes the AoI $\bar{\Delta}^{\pi}$.

Chapter 4

Randomized Policy

Let R denote a class of policies known as Randomized Policy in which the gateway polls a sensor or transmits updates to the monitor according to a probability mass function. While following a policy in R, the gateway chooses to poll sensor i with probability $p_i \in [0, 1]$ for $i \in \{1, 2, 3 \dots n\}$ and chooses to transmit the sensor updates to the monitor with probability p_0 such that $\sum_{i=1}^{n} p_i + p_0 = 1$. These probabilities do not change with time.

Any policy in R is fully characterized by the probabilities $p_0, p_1, \dots, p_n$ and does not require any other information, for example the ages of the sensors' updates at the gateway and the monitor, at any decision instant. We derive the expression of AoI for a randomized policy. We consider the cases of homogeneous and heterogeneous sensors. We demonstrate via simulations the sensitivity of average age to the choice of probabilities close to the age minimizing probability vector. Later in Chapter 7 we compare the randomized policy with others.

4.1 Deriving the Average Age

Figure 4.1 depicts all the random variables involved in the evolution of age at the monitor of any sensor i, when following a randomized policy. L_1 denotes the number of decisions made up to and including the first time sensor i is polled after its last age reset at the monitor. Therefore, L_1 is geometrically distributed with parameter p_i. The probability mass function (PMF) of L_1 is given by

$$P(L_1 = k) = (1 - p_i)^{k-1} p_i, k \geq 1. \tag{4.1}$$

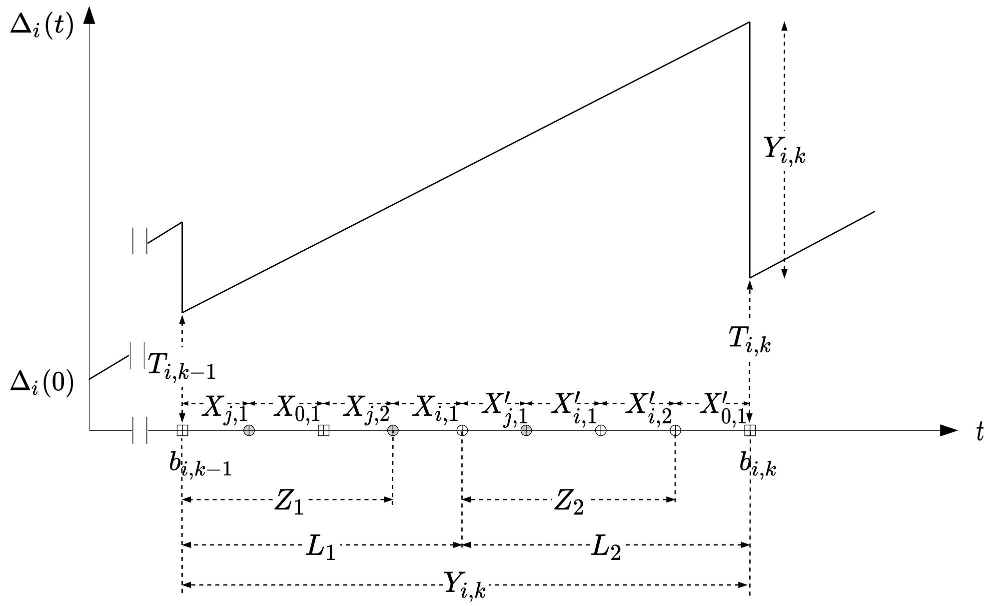


Figure 4.1: Sample function of age of sensor i at the monitor in Randomized policy. The empty circles are time instants when a fresh update from i was received by the gateway. The filled circles correspond to the same for another sensor j. The empty squares are when the monitor receives a transmission from the gateway.

We know from the properties of the geometric distribution that $E[L_1] = 1/p_i$ and $\text{Var}[L_1] = (1-p_i)/p_i^2$ where $E[.]$ and $\text{Var}[.]$ are the expectation and the variance operators respectively. Let $L_1 - 1$ decision slots constitute a total transmission length denoted by Z_1. Let $s_{j,k}$ be the number of pollings of sensors $1 \leq j \leq n, j \neq i$ in the interval of length Z_1. We have $s_{j,k} \geq 0$. Let $s_{0,k}$ be the number of transmissions to the monitor by the gateway in the interval Z_1. We have $s_{0,k} \geq 0$. The length of interval

$$Z_1 = \sum_{\substack{j=1 \\ j \neq i}}^{n} \sum_{l=1}^{s_{j,k}} X_{j,l} + \sum_{l=1}^{s_{0,k}} X_{0,l}. \tag{4.2}$$

Similarly, L_2 denotes the number of decisions made after L_1 slots, till the instant age of sensor i undergoes a reset at the monitor again. Therefore, L_2 is geometrically distributed with parameter p_0. Its PMF is given by

$$P(L_2 = k) = (1-p_0)^{k-1} p_0, k \geq 1. \tag{4.3}$$

Again, $E[L_2] = 1/p_0$ and $\text{Var}[L_2] = (1-p_0)/p_0^2$.

Let $L_2 - 1$ decision slots correspond to a time interval of length Z_2. Let $s'_{j,k}$ be the number of pollings of sensors $1 \leq j \leq n$ in the interval of length Z_2. We have

$s'_{j,k} \geq 0$. Note that there can be no transmission to the monitor during Z_2.

$$Z_2 = \sum_{j=1}^{n} \sum_{l=1}^{s'_{j,k}} X'_{j,l}. \tag{4.4}$$

We can now express $Y_{i,k}$ in terms of variables defined above.

$$Y_{i,k} = Z_1 + X_{i,1} + Z_2 + X'_{0,1}. \tag{4.5}$$

To calculate average age (3.4), we must derive $E[Y]$, $E[Y^2]$, and $E[YT]$. We provide a summary next. The details can be found in Appendix A. We have

$$E[Y_{i,k}] = \left(\frac{1}{p_0} + \frac{1}{p_i}\right) \sum_{j=0}^{n} p_j E[X_j] \tag{4.6}$$

$$E[Y_{i,k}^2] = \left(\frac{1}{p_0} + \frac{1}{p_i}\right) E_j^2 + \left(\frac{2}{p_0^2} + \frac{2}{p_i^2} + \frac{2}{p_0 p_i}\right) E_{jj'} - \left(\frac{2E[X_0]}{p_0} + \frac{2E[X_i]}{p_i}\right) E_j, \tag{4.7}$$

where,

$$E_j^2 = \sum_{j=0}^{n} p_j E[X_j^2], \tag{4.8a}$$

$$E_{jj'} = \sum_{j=0}^{n} \sum_{j'=0}^{n} p_j p_{j'} E[X_j] E[X_{j'}], \tag{4.8b}$$

$$E_j = \sum_{j=0}^{n} p_j E[X_j]. \tag{4.8c}$$

Note that $Y_{i,k}$ and $T_{i,k-1}$ are independent random variables. This is simply because each is a random sum of random variables (polling times and transmission times to the monitor) that are mutually independent. Given this, we can simplify and write $E[Y_{i,k} T_{i,k-1}] = E[Y_{i,k}] E[T_{i,k-1}]$. Thus, we only need to find $E[T_{i,k-1}]$ to be able to calculate average age.

We can write $T_{i,k-1}$ as

$$T_{i,k-1} = X_{i,1} + Z + X'_{0,1}, \tag{4.9}$$

where Z constitutes the total transmission length in which sensors $j \neq i$ get polled and there is no transmission to the monitor. The derivation for $E[T_{i,k-1}]$ can be found in Appendix A and is equal to

$$E[T_{i,k-1}] = E[X_0] + \frac{1}{p_0 + p_i} \sum_{\substack{j \neq 0 \\ j \neq i}} p_j E[X_j] + E[X_i] \tag{4.10}$$

Substituting (4.6),(4.7),(4.10) in (3.4), the average age equation for sensor i is obtained as

$$\bar{\Delta}_i^R = \frac{E_j^2}{2E_j} + \left(\frac{1}{p_0} + \frac{1}{p_i} - \frac{1}{p_0 + p_i}\right) \frac{E_{jj'}}{E_j} + \left(\frac{1}{p_0 + p_i}\right) E_j. \tag{4.11}$$

This gives us the time average age the of system as

$$\bar{\Delta}^R = \frac{E_j^2}{2E_j} + \frac{E_{jj'}}{E_j} \frac{1}{n} \sum_{i=1}^{n} \left(\frac{1}{p_0} + \frac{1}{p_i} - \frac{1}{p_0 + p_i}\right) + \frac{E_j}{n} \sum_{i=1}^{n} \frac{1}{p_0 + p_i}. \tag{4.12}$$

We observe that $E_{jj'} = (E_j)^2$. Therefore,

$$\bar{\Delta}^R = \frac{E_j^2}{2E_j} + \frac{E_j}{n} \sum_{i=1}^{n} \frac{1}{p_i} + \frac{E_j}{p_0}. \tag{4.13}$$

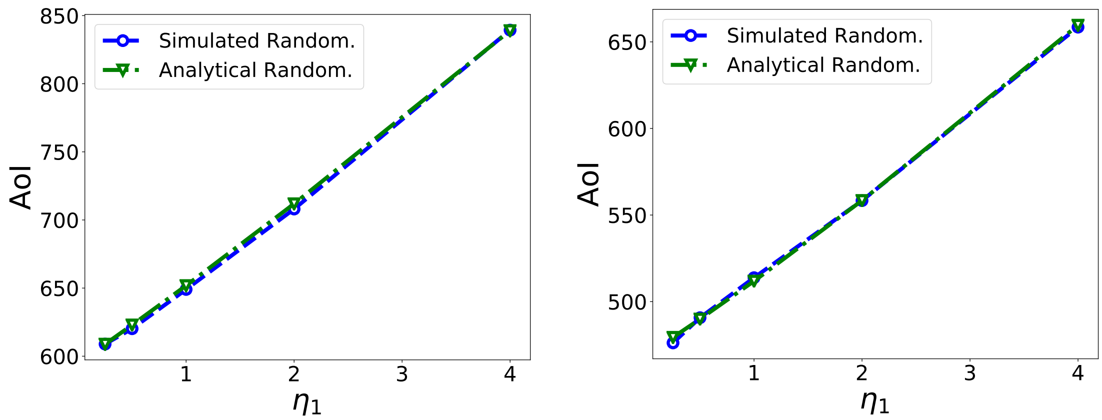
Finally, let us define a network parameter η_1 as

$$\eta_1 = \frac{E[X_0]}{\sum\limits_{i=1}^{n} E[X_i]/n} \tag{4.14}$$

Notice, as the transmission time to monitor becomes larger as compared to polling times of the sensors, η_1 also becomes larger. Figure 4.2 compares the average age of the network given by the analytical expression in (4.13) with that obtained using simulations for different values of η_1. Simulation value for each η_1 was obtained after averaging the AoI over 10 iterations where each iteration was long enough to ensure a minimum of 1500 resets in age of every sensor at the monitor.

4.2 Optimal Randomized Policy for Homogeneous Sensors

Recall that homogeneous sensors have iid polling times. As always, the transmission time to the monitor is independent of the polling times but maybe non-identical. In optimal Randomized policy for homogeneous sensors, one would expect that the gateway does not poll a certain sensor more often than the others. We assume $p_i^{opt} = p^{opt}$, for all sensors $i \in \{1, 2, 3 \ldots n\}$. We are unable to find a closed form analytical solution to the optimal probabilities for any selection of expected values and variances of sensor polling times and transmission time to the monitor. We use the MATLAB function `fmincon` to find p^{opt} and p_0^{opt}.



(a) **(b)**

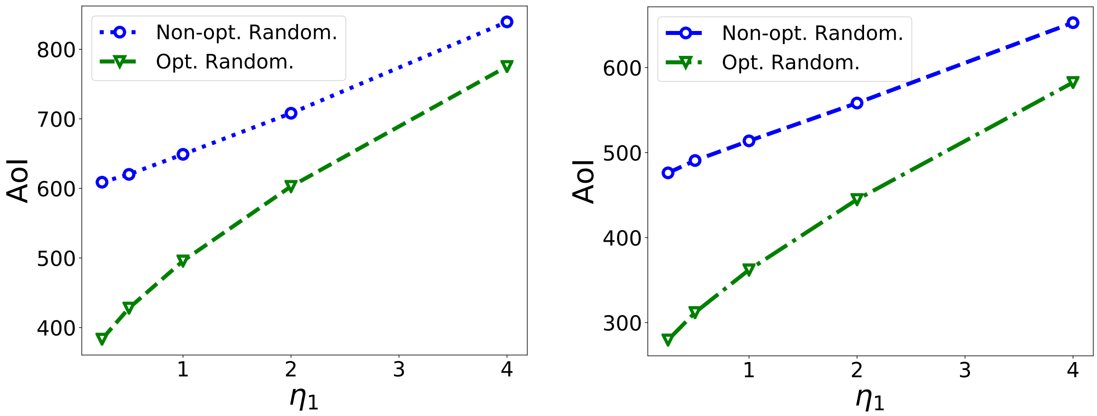
Figure 4.2: Average age obtained for $n = 10$ sensors and $p_i = \frac{1}{n+1}$, $i \in \{0, 1, 2, 3 \ldots n\}$. (a) Heterogeneous sensors where the polling times are uniformly distributed, (b) Heterogeneous sensors where the polling times are chosen as non-identical exponentially distributed random variables. The mean for each sensor is selected uniformly and randomly from $(0, 50)$.

However, it is instructive to consider the case when $\text{Var}[X] = 0$ (sensor polling times are deterministic). Minimizing average age (4.13) and using the fact that $\sum_{i=0}^{n} p_i = p_0^{opt} + np^{opt} = 1$ we obtain

$$p_0^{opt} = \frac{1}{\sqrt{n\eta_1} + 1}, \tag{4.15a}$$

$$p^{opt} = \frac{\sqrt{n\eta_1}}{n} p_0^{opt}. \tag{4.15b}$$

where using (4.14) $\eta_1 = \frac{E[X_0]}{E[X]}$. Interestingly, p_0^{opt} only depends on η_1, which is the ratio of expected value of transmission time to the monitor to the expected value of sensor polling time. p^{opt} is independent of individual expected values as long as their ratio remains the same. Note that the number of slots between two transmissions to the monitor is geometrically distributed with parameter p_0^{opt}. Thus the average number of such slots is $\sqrt{n\eta_1} + 1$. Curiously, for the Poll-s policy proposed in [1], the optimal value was found to be $s = \sqrt{n\eta_1}$, where s is the fixed number of sensors polled before transmitting to the gateway. Interestingly, for both the optimal randomized and optimal Poll-s, the average number of polling slots before sending to the monitor grow as $\sqrt{n}$.



(a) **(b)**

Figure 4.3: Average age obtained for $n = 10$ sensors with $p_i = \frac{1}{n+1}$, $i \in \{0, 1, 2, 3 \dots n\}$ and p_{opt}. (a) Polling times are uniformly distributed, (b) Polling times are exponentially distributed. However, non-identical for the sensors. The mean for each sensor is selected uniformly and randomly from $(0, 50)$.

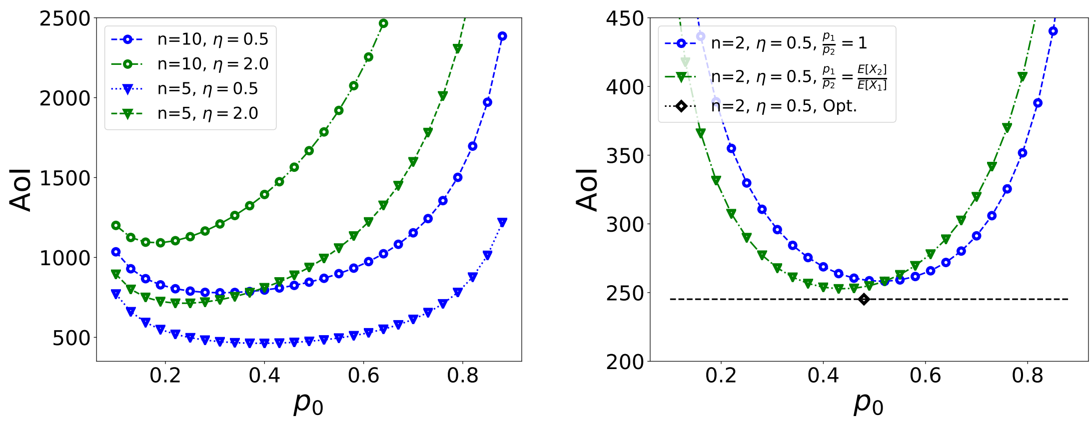
4.3 Optimal Randomized Policy for Heterogeneous Sensors

As before, the optimal probability vector p_i^{opt} for $i \in \{0, 1, 2, 3 \dots n\}$, that minimizes average age of system among other policies in R, depends on first and second moment of sensor polling times and monitor transmission time. Note that since the sensors are heterogeneous we would expect sensors to be polled with different probabilities. This results in $n+1$ variables of optimization in contrast to just two variables in the homogeneous case. We are unable to obtain an analytical solution for the optimal probabilities. We use MATLAB function `fmincon` to find p_i^{opt}, for all i.

In both the cases of homogeneous and heterogeneous sensors, as η_1 increases, we observe that probability p_0 decreases. This tells us that as cost of sending updates to monitor increases, we should send updates to monitor less often and instead keep polling sensors.

Figure 4.3, compares the AoI obtained for a selection of equal probabilities of polling of any sensor and sending to the monitor with optimal selections of the probabilities. Not surprisingly, the optimal probability vector results in lower AoI. In fact, using equal probabilities performs a lot worse.

Lastly, how sensitive AOI is to changes in the optimal probabilities. For this,



(a) **(b)**

Figure 4.4: Age of the network as a function of p_0. (a) Polling times are iid and uniformly distributed. (b) Polling times uniformly distributed. However, non-identical for the sensors.

we varied p_0 from 0.1 to 0.9 and plotted the corresponding AoI. From Figure 4.4a, we observe that small displacement in values of p_0 around point of minimum AoI achievable by this policy does not drastically impact performance. In Figure 4.4b where $n = 2$, the green curve gives minimum AoI not too far from the optimal value. These insights maybe useful when such a policy is implemented in real-life IoT networks where it may not be possible to implement the exact optimal probabilities.

Chapter 5

Other Scheduling Policies

5.1 Drift-based policy

Let us define a linear Lyapunov Function for minimizing the drift in gateway age when gateway polls a sensor as

$$L(\{\Delta^{'}_{j,t}\}_{j=1}^{n}) = L(t) = \frac{1}{n}\sum_{j=1}^{n}\Delta^{'}_{j,t}. \tag{5.1}$$

The Lyapunov Drift is defined as

$$\delta(S_t) = E[L(t+1) - L(t)|S_t], \tag{5.2}$$

where t is the decision instant after the current polling or transmission ends and $t+1$ is the next decision instant. Such a drift based policy in a single-hop broadcast setting has been looked at in [3]. The state of the network is $S_t = \{\Delta^{'}(t), \Delta(t)\}$. As $L(t)$ is a function of the age vector, it increases as average age of the network at time t increases. $\delta(S_t)$ represents the conditional expected increase in the average age that results from a decision. By minimizing the drift at each decision instant, we tend to keep the average age of the network small. Consider when a sensor i is polled. We have

$$\begin{aligned}\delta_i(S_t) &= \frac{1}{n}E[(\sum_{j=1}^{n}\Delta'_{j,t+1} - \sum_{j=1}^{n}\Delta'_{j,t})|S_t] \\ &= \frac{1}{n}E[(X_i - \Delta'_{i,t} + \sum_{j\neq i}X_i)|S_t] \\ &= \frac{1}{n}E[(nX_i - \Delta'_{i,t})|S_t] \\ &= \frac{1}{n}(nE[X_i] - \Delta'_{i,t}) \\ &= E[X_i] - \frac{\Delta'_{i,t}}{n}.\end{aligned} \tag{5.3}$$

Now let us minimize the drift $\delta_0(S_t)$ in the average age at the monitor when gateway transmits updates to the monitor

$$\delta_0(S_t) = \frac{1}{n}E[(\sum_{j=1}^{n}\Delta_{j,t+1} - \sum_{j=1}^{n}\Delta_{j,t})|S_t]. \tag{5.4}$$

At $t+1$, when monitor receives a transmission from the gateway, age of sensors at the monitor gets reset to age of sensors at the gateway. Therefore

$$\begin{aligned}\Delta_{t+1} &= \Delta'_{t+1} \\ &= \Delta'_t + X_0.\end{aligned} \tag{5.5}$$

Using above evolution of Δ_{t+1} in (5.4), we obtain

$$\begin{aligned}\delta_0(S_t) &= \frac{1}{n}E[\sum_{j=1}^{n}(\Delta_{j,t+1} - \Delta_{j,t})|S_t] \\ &= \frac{1}{n}E[\sum_{j=1}^{n}(\Delta'_{j,t} + X_0 - \Delta_{j,t})|S_t] \\ &= \frac{1}{n}E[(nX_0 + \sum_{j=1}^{n}(\Delta'_{j,t} - \Delta_{j,t}))|S_t] \\ &= \frac{1}{n}(nE[X_0] + \sum_{j=1}^{n}(\Delta'_{j,t} - \Delta_{j,t})) \\ &= E[X_0] + \frac{1}{n}\sum_{j=1}^{n}(\Delta'_{j,t} - \Delta_{j,t}).\end{aligned} \tag{5.6}$$

For minimizing drift, at decision instant t the gateway chooses an action given by

$$\min\left(\{\delta_j(S_t)\}_{j=1}^{n},\ \delta_0(S_t)\right). \tag{5.7}$$

Before making any observations let us define two policies given in [1]. One, a policy named $\widehat{s^*}$,MAF was proposed. In this, the gateway polls a fixed number of sensors s before sending updates to the monitor, following Maximum Age First scheduling which is in fact optimal when sensors are homogeneous. Second, a policy named $\widehat{s^*}$,MCA (Minimum Change in Age) is proposed when sensors are heterogeneous. This policy also polls s sensors before transmitting polled updates to the monitor and while not transmitting to the monitor, chooses to poll the sensor that minimizes the conditional expected change in average age at the gateway.

We observe that the above Drift-based policy is similar to policy $\widehat{s^*}$,MCA [1] while determining which sensor to poll. The basic difference is that Drift-based policy does not poll fixed number of sensors s before transmitting to the monitor. The decision of transmitting to the monitor is also determined based on the drift as calculated in (5.6). On the other hand, $\widehat{s^*}$,MCA doesn't requires drift to be calculated for the monitor.

5.2 Scheduling policy based on Sensor Polling Times

The optimal value of s was shown to be $\approx$ round($\sqrt{\eta_1 n}$) for the policy $\widehat{s^*}$,MAF, when the sensors are homogeneous. We propose a novel policy that uses the same value of s but the polling schedule of sensors is determined by their expected polling times. Let us call this policy as $\widehat{s^*}$,Ordered.

In this work we will only consider cases where n is an integer multiple of s. It means the total number of sensors n can be polled in m groups where $m = \frac{n}{s}$ is an integer. Further, as in the case of MAF, we will restrict ourselves to not polling a sensor again before all other sensors have been polled. This results in m groups of s unique sensors, wherein a group sees s unique sensors polled followed by a transmission to the monitor. Also, sensors in different groups are unique too. As a result, for any sensor i,

$$E[Y_i] = E[Y] = \sum_{j=1}^{n} E[X_j] + mE[X_0]. \tag{5.8}$$

Therefore, AoI for $\widehat{s^*}$,Ordered only depends on $E[T_i]$ of every sensors i. This policy attempts to minimize average age by: scheduling s sensors between every two consecutive transmissions to the monitor in the decreasing order of their polling times and keeping the sum $\sum_{i=1}^{n} E[T_i]$ minimum. Let (j, k) define the position of a sensor in a schedule, where $1 \leq j \leq s$ and $1 \leq k \leq m$. The sensor at the $(j,k)^{\text{th}}$ position is at the j^{th} position in the k^{th} group of sensors.

Theorem 1. *Given n and s for which $m \in \mathbb{Z}_{>0}$. Consider the sequence*

$$\begin{gathered}(1,1),(2,1),\dots(s,1),0,\\ \vdots\\ (1,r),(2,r)\dots(s,r),0,\\ \vdots\\ (1,m),(2,m)\dots(s,m),0.\end{gathered}$$

that repeats itself. The optimal sequence satisfies $E[X_{(j,k)}] \geq E[X_{(j',k')}]$ when $j < j'$, for all $1 \leq k, k' \leq m$.

Proof. Let the sequence of polling be denoted as in the theorem. For the purposes of this proof we will abuse notation and let $X_{(j,k)}$ to be the polling time of the sensor in position (j,k) of the given schedule. We continue to use 0 to denote the monitor. We can write

$$\begin{aligned}\sum_{i=1}^{n} E[T_i] &= \sum_{k=1}^{m}\sum_{j=1}^{s} jE[X_{(j,k)}] + mE[X_0]\\ &= s\left(\sum_{k=1}^{m} E[X_{(s,k)}]\right) + \dots + j\left(\sum_{k=1}^{m} E[X_{(j,k)}]\right) + \dots \\ &\quad \dots + 1\left(\sum_{k=1}^{m} E[X_{(1,k)}]\right) + mE[X_0]\end{aligned} \tag{5.9}$$

Consider the following sets

$$\begin{aligned}\mathcal{X}_1 &= \Big\{E[X_{(1,1)}],\ E[X_{(1,2)}],\dots E[X_{(1,m)}]\Big\},\\ \mathcal{X}_i &= \Big\{E[X_{(i,1)}],\ E[X_{(i,2)}],\dots E[X_{(i,m)}]\Big\},\\ &\vdots\\ \mathcal{X}_s &= \Big\{E[X_{(s,1)}],\ E[X_{(s,2)}],\dots E[X_{(s,m)}]\Big\}.\end{aligned}$$

Let us define an operation $\mathcal{X}_i^+$ which calculates the sum of elements in $\mathcal{X}_i$.

For the given sequence, define the cost

$$C \triangleq \mathcal{X}_1^+ + \dots + j\mathcal{X}_j^+ + \dots + j^{'}\mathcal{X}_{j^{'}}^+ + \dots + s\mathcal{X}_s^+. \tag{5.11}$$

Pick indices j, j' such that $j < j'$. Suppose there exist positions (j, k) and (j', k') in the schedule, such that $E[X_{(j,k)}] < E[X_{(j',k')}]$, for any k, k'. We claim that the schedule that is obtained by swapping the sensors at positions (j, k) and (j', k') has a smaller cost C' (and thus smaller AoI) than the given schedule.

For the new schedule let the sets corresponding to $\mathcal{X}_i$ be denoted by $\tilde{\mathcal{X}}_i$. We can now write the cost

$$C' \triangleq \mathcal{X}_1^+ + \cdots + j\tilde{\mathcal{X}}_j^+ + \cdots + j'\tilde{\mathcal{X}}_{j'}^+ + \cdots + s\mathcal{X}_s^+ \tag{5.12}$$

Subtracting equation (5.12) from equation (5.11), we get

$$\begin{aligned} C - C' &= j(\mathcal{X}_j^+ - \tilde{\mathcal{X}}_j^+) + j'(\mathcal{X}_{j'}^+ - \tilde{\mathcal{X}}_{j'}^+) \\ &= j(E[X_{j,k}] - E[X_{j',k'}]) + j'(E[X_{j',k'}] - E[X_{j,k}]) \\ &= (j' - j)(E[X_{j',k'}] - E[X_{j,k}]) \\ &> 0 \end{aligned} \tag{5.13}$$

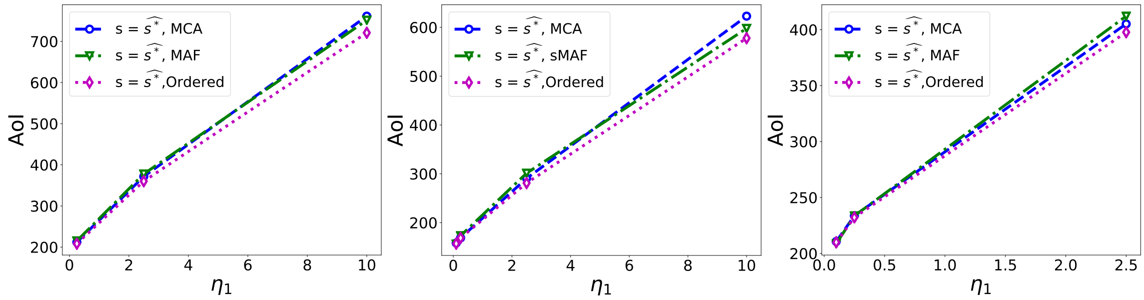
For any given scheduling, we can continue to swap and reduce cost till no more indices satisfy the condition for swapping. The resulting schedule will be the optimal schedule. ■

Now consider the round robin policy. In this policy, gateway polls every sensor exactly once before the updates are sent to the monitor. The following is true.

Corollary 1.1. *Given $s = n$ and $E[X_1] \geq E[X_2] \geq \ldots \cdots \geq E[X_n]$, the optimal sequence for Round Robin is $i_1^* = 1,\ i_2^* = 2, \ldots i_n^* = n$.*

In the round robin policy, i_1^* denotes the index of the sensor that is polled soon after and sensor with index i_n^* is polled immediately before transmission to the monitor.

We simulated the $\widehat{s^*}$,Ordered for those specific values of η_1 for which $n \ \mathbf{mod}\ s = 0$ and results are shown in Figure 5.1. As we expected, scheduling sensors on the basis of their expected polling times leads to a better AoI performance. Note that the policy outperforms even $\widehat{s^*}$,MCA which was proposed for heterogeneous sensors. Another added advantage of $\widehat{s^*}$,Ordered over other policies is that it does not require the knowledge of state of the network for making scheduling decisions. Therefore, computations at the gateway are avoided.



(a) **(b)** **(c)**

Figure 5.1: Comparison for $\widehat{s^*}$,Ordered. Average age obtained for $n = 10$ sensors. (a) Polling times are uniformly distributed, (b) Polling times are exponentially distributed and (c) Polling times are hyper-exponentially distributed. However, non-identical for the sensors. The mean for each sensor is selected uniformly and randomly from $(0, 50)$.

As of now, we have restricted ourselves in looking at cases where n is an integer multiple of s or when n is not a prime. When either of these two cases is not true, finding a policy which minimizes age is a scope for future work where scheduling is based on expected polling times.

Chapter 6

Lower Bound on AoI

In this chapter, we derive a lower bound on achievable AoI. Consider a sample function of the age process $\Delta_i(t)$ for sensor i (Figure 3.2) over a finite time horizon T. Let $D_i(T)$ denote the number of times the age of sensor i gets reset at the monitor over the time interval T. Let $\gamma_i(T),\ i \in 1, 2, \ldots, n$, denote the number of times the gateway polls sensor i and $\gamma_0(T)$ be the number of times gateway transmits updates to the monitor. $Y_{i,k}$, as defined earlier, is the time between $k-1^{th}$ and k^{th} reset of age of sensor i at monitor. The time interval T can be expressed as

$$T = \sum_{j=1}^{D_i(T)} Y_{i,j} + \tau \geq \sum_{i=1}^{n} \sum_{j=1}^{\gamma_i(T)} X_{i,j} + \sum_{j=1}^{\gamma_0(T)} X_{0,j} \tag{6.1}$$

Here $\tau \geq 0$ is the time that elapses between the last reset in age of sensor i at the monitor and the end of the interval T. In order to write the average age of sensor i for the above sample function over the finite interval, we define the sample mean and the sample variance operators by $M[.]$, $V[.]$, respectively. We have

$$M[X_i] = \frac{\sum_{k=1}^{\gamma_i(T)} X_{i,k}}{\gamma_i(T)}, \tag{6.2a}$$

$$M[Y_i] = \frac{\sum_{k=1}^{D_i(T)} Y_{i,k}}{D_i(T)}, \tag{6.2b}$$

$$M[Y_i^2] = \frac{\sum_{k=1}^{D_i(T)} Y_{i,k}^2}{D_i(T)}, \tag{6.2c}$$

$$M[Y_i T_{i,k-1}] = \frac{\sum_{k=1}^{D_i(T)} Y_{i,k} T_{i,k-1}}{D_i(T)}. \tag{6.2d}$$

Using (6.2) to write the AoI Δ_T^π for the sample function, we have,

$$\begin{aligned} \Delta_T^\pi &= \frac{1}{n}\sum_{i=1}^{n}\left[\frac{M[Y_i T_{i,k-1}]}{M[Y_i]} + \frac{M[Y_i^2]}{2M[Y_i]}\right] \\ &= \frac{1}{n}\sum_{i=1}^{n}\left[\frac{M[Y_i T_{i,k-1}]}{M[Y_i]} + \frac{V[Y_i] + M[Y_i]^2}{2M[Y_i]}\right] \end{aligned} \tag{6.3}$$

Let us now manipulate the expression of average age of the system as follows to obtain a lower bound.

$$\begin{aligned} \Delta_T^\pi &= \frac{1}{n}\sum_{i=1}^{n}\left[\frac{V[Y_i] + M[Y_i]^2}{2M[Y_i]} + \frac{M[Y_i T_{i,k-1}]}{M[Y_i]}\right] \\ &\overset{(a)}{\geq} \frac{1}{n}\sum_{i=1}^{n}\left[\frac{M[Y_i]}{2} + \frac{M[Y_i T_{i,k-1}]}{M[Y_i]}\right] \\ &\overset{(b)}{\geq} \frac{1}{n}\sum_{i=1}^{n}\left[\frac{M[Y_i]}{2} + M[X_i] + M[X_0]\right] \\ &\overset{(c)}{=} \frac{1}{2n}\left(\sum_{i=1}^{n} M[Y_i] + 2\sum_{i=1}^{n}(M[X_i] + M[X_0])\right) \\ &\overset{(d)}{\geq} \frac{1}{2n}\left(\sum_{i=1}^{n}\sum_{j=1}^{\gamma_i(T)} X_{i,j} + \sum_{j=1}^{\gamma_0(T)} X_{0,j}\right)\sum_{i=1}^{n}\frac{1}{D_i(T)} + \frac{1}{n}\sum_{i=1}^{n}(M[X_i] + M[X_0]) \\ &\overset{(e)}{=} \frac{1}{2n}\left(\sum_{i=1}^{n} M[X_i]\gamma_i(T) + M[X_0]\gamma_0(T)\right)\sum_{i=1}^{n}\frac{1}{D_i(T)} + \frac{1}{n}\sum_{i=1}^{n}(M[X_i] + M[X_0]) \\ &\overset{(f)}{=} \frac{1}{2n}\left(\sum_{i=1}^{n} M[X_i]\gamma_i(T)\sum_{i=1}^{n}\frac{1}{D_i(T)} + M[X_0]\gamma_0(T)\sum_{j=1}^{n}\frac{1}{D_j(T)}\right) \\ &+ \frac{1}{n}\sum_{i=1}^{n}(M[X_i] + M[X_0]) \\ &\overset{(g)}{\geq} \frac{1}{2n}\left(\left(\sum_{i=1}^{n}\sqrt{\frac{M[X_i]\gamma_i(T)}{D_i(T)}}\right)^2 + M[X_0]\left(\frac{\gamma_0(T)}{D_1(T)} + \frac{\gamma_0(T)}{D_2(T)} + \cdots + \frac{\gamma_0(T)}{D_n(T)}\right)\right) \\ &+ \frac{1}{n}\sum_{i=1}^{n}(M[X_i] + M[X_0]) \end{aligned} \tag{6.4}$$

where (a) uses the Jensen's inequality to establish that $V[Y_i] \geq 0$, (d) uses (6.2b) followed by inequality in (6.1), (e) uses (6.2a) and (g) uses the Cauchy-Schwarz inequality.

Inequality (b) is obtained by using the fact that $T_{i,k-1} \geq X_i + X_0$, and therefore $M[Y_i T_{i,k-1}] \geq M[Y_i(X_i + X_0)] = M[Y_i]M[(X_i + X_0)]$. Note that we will also obtain a lower bound by simply ignoring $M[Y_i T_{i,k-1}] \geq 0$. However, empirically we have observed that this leads to a rather loose bound.

All the above steps except (b) follow the steps used in deriving the lower bound for a single-hop broadcast network in [3].

Generally, every polling of sensor i will not lead to a reset in its age at the monitor, that is, $\frac{\gamma_i(T)}{D_i(T)} \geq 1$. Alternatively, we can say that the number of resets in age of sensor i at the monitor cannot be greater than the number of times the gateway polls it. Also, $\frac{\gamma_0(T)}{D_i(T)} \geq 1$ since age of sensor i may not get reset every time gateway transmits the to monitor. Sensor i may not be polled by the gateway for fresh updates before transmitting to the monitor. For deriving the lower bound, we use the lowest value these ratios can take, that is, 1. Finally, taking limit $T \to \infty$ in (6.4) followed by expectation over any policy in $\pi \in \Pi$, we get

$$\begin{aligned} E\left[\lim_{T\to\infty} \Delta_T^\pi\right] &\geq E\left[\lim_{T\to\infty} \frac{1}{2n}\left(\left(\sum_{i=1}^{n}\sqrt{\frac{M[X_i]\gamma_i(T)}{D_i(T)}}\right)^2 + M[X_0]\left(\frac{\gamma_0(T)}{D_1(T)} + \cdots + \frac{\gamma_0(T)}{D_n(T)}\right)\right)\right. \\ &\quad \left. + \frac{1}{n}\sum_{i=1}^{n}(M[X_i] + M[X_0])\right] \\ &\geq \frac{1}{2n}\left(\left(\sum_{i=1}^{n}\sqrt{E[X_i]}\right)^2 + nE[X_0]\right) + \frac{1}{n}\sum_{i=1}^{n}(E[X_i] + E[X_0]) \\ &= LB \end{aligned} \tag{6.5}$$

Along the lines in [3], in which authors use Fatou's lemma to establish the lower bound, the expression obtained in (6.5) is the lower bound to our optimization problem. Specifically, by Fatou's lemma, the average age for any policy π, $\lim_{T\to\infty} E[\Delta_T^\pi] \geq E\left[\lim_{T\to\infty} \Delta_T^\pi\right] \geq LB$. The first inequalities is valid since $\Delta_T^\pi > 0$. The second is simply given by (6.5).

Chapter 7

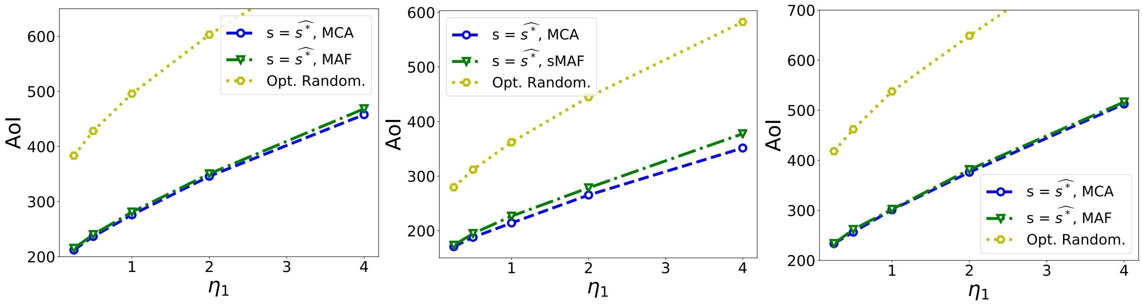
Results

In this section, we evaluate the performance of the four policies analyzed in this work: i) Randomized policy; ii) Drift-based policy; iii) $\widehat{s^*}$,Ordered and iv) Round-Robin policy in terms of AoI. We simulated these policies for different distributions like uniform, exponential and hyper-exponential and compared their performance with $\widehat{s^*}$,MAF and $\widehat{s^*}$,MCA proposed in [1].

Since Randomized policy does not leverage the knowledge of state of the network, it is expected that its performance will be worse than the other policies we have considered. This is seen in Figure 7.1 for different distributions. That said the policy doesn't require any current state information to make a decision.

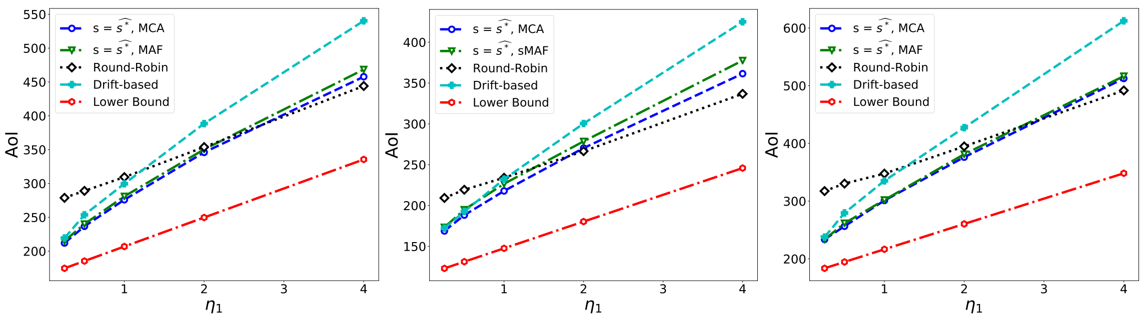
From Figure 7.2, we observe that for heterogeneous network the drift-based performs worse than $\widehat{s^*}$,MCA and $\widehat{s^*}$,MAF. Figure 7.2 also shows that Round-Robin policy gives lower age when compared to policy such as $\widehat{s^*}$,MCA and $\widehat{s^*}$,MAF beyond a certain η_1 for which $s \leq 10$. Therefore, Round Robin is a fair policy to use when η_1 is large for a given n. It should be preferred over $\widehat{s^*}$,MCA since the former does not require the current state values of the network and the latter requires the gateway age and therefore carries out computation every time the gateway has to make a decision.

Lastly, Figure 7.3 shows the performance of all policies when compared with the lower bound. The Randomized policy is within about 2.5 times the lower bound while other policies are well within 2 times the lower bound. Given the simplicity of the implementation of the randomized policy, it maybe desirable to use it over proposed policies.



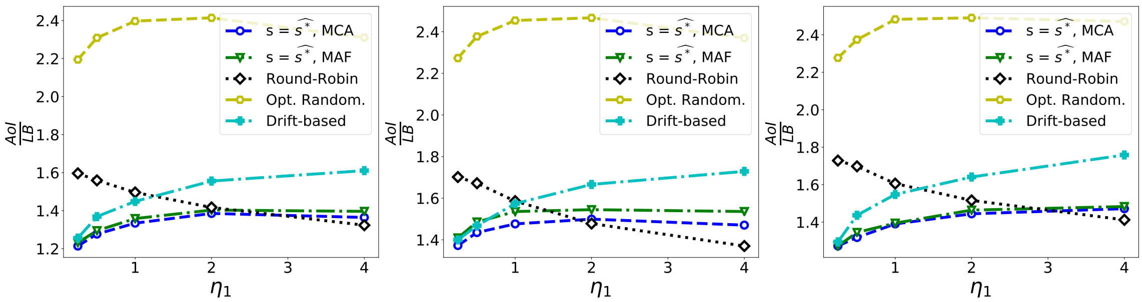
(a) **(b)** **(c)**

Figure 7.1: Average age obtained for $n = 10$ sensors. (a) Polling times are uniformly distributed, (b) Polling times are exponentially distributed and (c) Polling times are hyper-exponentially distributed. However, non-identical for the sensors. The mean for each sensor is selected uniformly and randomly from $(0, 50)$.



(a) **(b)** **(c)**

Figure 7.2: Comparison of average AoI for all policies obtained for $n = 10$ sensors. (a) Polling times are uniformly distributed, (b) Polling times are exponentially distributed and (c) Polling times are hyper-exponentially distributed. However, non-identical for the sensors. The mean for each sensor is selected uniformly and randomly from $(0, 50)$.



(a) **(b)** **(c)**

Figure 7.3: Policy comparison wrt lower bound. Average age obtained for$n = 10$ sensors. (a) Polling times are uniformly distributed, (b) Polling times are exponentially distributed and (c) Polling times are hyper-exponentially distributed. However, non-identical for the sensors. The mean for each sensor is selected uniformly and randomly from $(0, 50)$.

Chapter 8

Summary

In this thesis, we focused on a two-hop network as opposed to analyzing a more common single-hop network. We addressed the problem of minimizing AoI in a gateway based network. We developed an optimal Randomized policy whose performance was evaluated both analytically and through simulation. We also developed a novel policy called s,Ordered policy. It showed improvement over existing policies while using no information about the network state. We developed a drift-based policy using the concept of Lyapunov drift. This policy in spite of using information about state of the network showed no improvement over existing policies. Lastly, we also derived a lower bound to the achievable AoI. In spite of its simplicity, optimal Randomized policy performed within $2.5\times$ the lower bound. The performances of all other policies were well within $2\times$ the lower bound. A general s,Ordered policy is a possible extension to this work.

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Appendix A

Derivation for Randomized policy

A.1 Derivation of $E[Y_i]$

From 4.1, we already know $E[Y_i] = E[Z_1] + E[X_i] + E[Z_2] + E[X_0]$. We need to find expectation of Z_1 and Z_2. Now, Z_1 is the interval of time before sensor i gets polled the very first time. Therefore,

$$Z_1 = \sum_{\substack{j=0 \\ j \neq i}}^{n} \sum_{k=1}^{s_j} X_{j,k} \tag{A.1}$$

where s_j is the number of times sensor j got polled during Z_1. Also, $\sum_{\substack{j=0 \\ j \neq i}}^{n} s_j = L_1 - 1$, where s_j has multinomial distribution. Further, taking expectation in (A.1) we have

$$E[Z_1] = \sum_{j \neq i} E\left[\sum_{k=1}^{s_j} X_{j,k}\right] \tag{A.2}$$

Using the method of iterated expectation to find $E[Z_1]$, we can write

$$
\begin{aligned}
E[Z_1] &= E_{s_j}\left[E[Z_1|s_{j,j\neq i}]\right] \\
&= \sum_{j\neq i} E\left[E\left[\sum_{k=1}^{s_j} X_{j,k}|s_j\right]\right] \\
&= \sum_{j\neq i} E\left[\sum_{k=1}^{s_j} E\left[X_{j,k}|s_j\right]\right] \\
&= \sum_{j\neq i} E\left[\sum_{k=1}^{s_j} E\left[X_{j,k}\right]\right] \\
&= \sum_{j\neq i} E\left[\sum_{k=1}^{s_j} E\left[X_j\right]\right] \\
&= \sum_{j\neq i} E\left[s_j E\left[X_{j,k}\right]\right] \\
&= \sum_{j\neq i} E[s_j]E[X_j]
\end{aligned}
\tag{A.3}
$$

where, the third equality results from the fact that $X_{j,k}$ is independent of s_j and the fourth equality from the fact that $X_{j,k} \sim f_{X_j}$. Let us find $E[s_j]$. Note that given L_1, random variable s_j is a multinomial coefficient with parameter $q_j = \frac{p_j}{1-p_i}$.

$$
\begin{aligned}
E[s_j] &= E_{L_1}\left[E[s_j|L_1]\right] \\
&= E\left[(L_1-1)\frac{p_j}{\sum\limits_{k\neq i} p_k}\right] \\
&= E\left[(L_1-1)\frac{p_j}{1-p_i}\right] \\
&= \frac{p_j}{1-p_i}E[L_1-1] \\
&= \frac{p_j}{1-p_i}\frac{1-p_i}{p_i} \\
&= \frac{p_j}{p_i}
\end{aligned}
\tag{A.4}
$$

where, in the first equality the inner conditional expectation is the expected number of times sensor $j, j \neq i$ is polled given L_1 slots. Finally,

$$
E[Z_1] = \sum_{\substack{j=0 \\ j\neq i}}^{n} \frac{p_j}{p_i} E[X_j] \tag{A.5}
$$

Similarly we can derive $E[Z_2]$ to get

$$
E[Z_2] = \sum_{j=1}^{n} \frac{p_j}{p_0} E[X_j] \tag{A.6}
$$

Using (A.5) and (A.6) to find $E[Y_i]$, we get,

$$E[Y_i] = \sum_{j=0}^{n} \left(\frac{1}{p_i} + \frac{1}{p_0}\right) p_j E[X_j] \frac{p_j}{p_0} E[X_j] \tag{A.7}$$

A.2 Derivation of $E[Y_i^2]$

We can write $E[Y_i^2]$ as

$$\begin{aligned} E[Y_i^2] = E[Z_1^2] + E[X_i^2] + E[Z_2^2] + E[X_0^2] \\ + 2(E[Z_1] + E[Z_2])(E[X_i] + E[X_0]) + 2E[Z_1]E[Z_2] + 2E[X_i]E[X_0] \end{aligned} \tag{A.8}$$

Given the similarity between Z_1 and Z_2, we only show the derivation for $E[Z_1^2]$.

$$\begin{aligned} E[Z_1^2] = \sum_{j \neq i} E\left[\sum_{k=1}^{s_j} X_{j,k}^2\right] + \sum_{j \neq i} \sum_{\substack{j' \neq i \\ j' \neq j}} E\left[\sum_{k=1}^{s_j} \sum_{k'=1}^{s_{j'}} X_{j,k} X_{j',k'}\right] \\ + \sum_{j \neq i} E\left[\sum_{k=1}^{s_j} \sum_{\substack{k'=1 \\ k' \neq k}}^{s_j} X_{j,k} X_{j,k'}\right] \end{aligned} \tag{A.9}$$

Consider $E\left[\sum_{k=1}^{s_j} X_{j,k}^2\right]$

$$\begin{aligned} E\left[\sum_{k=1}^{s_j} X_{j,k}^2\right] &= E_{s_j}\left[E\left[\sum_{k=1}^{s_j} X_{j,k}^2 | L_j\right]\right] \\ &= E\left[s_j E\left[X_j^2\right]\right] \\ &= E\left[s_j\right] E\left[X_j^2\right] \\ &= \frac{p_j}{p_i} E[X_j^2] \end{aligned} \tag{A.10}$$

where the last equality uses similar derivation steps for $E[s_j]$ in Appendix A.1. Consider $E\left[\sum_{k=1}^{s_j} \sum_{k'=1}^{s_{j'}} X_{j,k} X_{j',k'}\right]$

$$\begin{aligned} E\left[\sum_{k=1}^{s_j} \sum_{k'=1}^{s_{j'}} X_{j,k} X_{j',k'}\right] &= E_{s_j, s_{j'}}\left[E\left[\sum_{k=1}^{s_j} \sum_{k'=1}^{s_{j'}} X_{j,k} X_{j',k'} | s_j, s_{j'}\right]\right] \\ &= E\left[s_j s_{j'} E[X_j] E[X_{j'}]\right] \\ &= E\left[s_j s_{j'}\right] E[X_j] E[X_{j'}] \end{aligned} \tag{A.11}$$

We can write $E\left[s_j s_{j'}\right] = E_{L_1}\left[E[s_j s_{j'}|L_1]\right]$. Further,

$$
\begin{aligned}
E[s_j s_{j'}|L_1] &= E\left[E[s_j s_{j'}|s_j, L_1]|L_1\right] \\
&= E\left[s_j E[s_{j'}|s_j, L_1]|L_1\right] \\
&= E\left[s_j(L_1 - s_j - 1)\frac{q_{j'}}{1-q_j}|L_1\right] \\
&= \frac{q_{j'}}{1-q_j}(L_1 E[s_j] - E[s_j^2] - E[s_j]) \\
&= q_{j'} q_j (L_1^2 - 3L_1 + 2)
\end{aligned}
\tag{A.12}
$$

where, given L_1 and s_j, $s_{j'}$ is a multinomial coefficient with parameter $q_{j'} = \frac{p_{j'}}{1-p_i}$. Finally, we get $E\left[sjs_{j'}\right]$ as

$$
\begin{aligned}
E\left[s_j s_{j'}\right] &= q_{j'} q_j E[L_1^2 - 3L_1 + 2] \\
&= q_{j'} q_j \frac{2(1-p_i)^2}{p_i^2}
\end{aligned}
\tag{A.13}
$$

Consider $E\left[\sum_{k=1}^{s_j} \sum_{\substack{k'=1 \\ k' \neq k}}^{s_j} X_{j,k} X_{j,k'}\right]$

$$
\begin{aligned}
E\left[\sum_{k=1}^{s_j} \sum_{\substack{k'=1 \\ k' \neq k}}^{s_j} X_{j,k} X_{j,k'}\right] &= E_{s_j}\left[E\left[\sum_{k=1}^{s_j} \sum_{\substack{k'=1 \\ k' \neq k}}^{s_j} X_{j,k} X_{j,k'}|s_j\right]\right] \\
&= E[s_j(s_j-1)]E[X_j]^2 \\
&= 2\frac{p_j^2}{p_i^2}\ E[X_j]^2
\end{aligned}
\tag{A.14}
$$

A.3 Derivation of $E[T_i]$

Consider the interval Z in which gateway never polls sensor i and does not transmit to the monitor. Let there be a total of $L-1$ pollings in the interval Z. Then L is geometrically distributed with parameter $(p_i + p_0)$.

$$
P[L = l] = (1 - (p_i + p_0))^{l-1}(p_i + p_0) \tag{A.15}
$$

Then $E[L] = \frac{1}{p_i + p_0}$. Note that,

$$
Z = \sum_{\substack{j \neq 0 \\ j \neq i}} \sum_{k=1}^{s_j} X_{j,k} \tag{A.16}
$$

where, s_j is the number of times sensor j got polled during Z. Also, $\sum\limits_{\substack{j\neq 0\\ j\neq i}} s_j = L-1$.

Further,

$$E[Z] = \sum_{\substack{j\neq 0\\ j\neq i}} E\left[\sum_{k=1}^{s_j} X_{j,k}\right] \tag{A.17}$$

$$\begin{aligned}
E[Z] &= E_{s_j}\left[E[Z|s_{j,j\neq i,0}]\right] \\
&= \sum_{\substack{j\neq 0\\ j\neq i}} E\left[E\left[\sum_{k=1}^{s_j} X_{j,k}|s_j\right]\right] \\
&= \sum_{\substack{j\neq 0\\ j\neq i}} E\left[\sum_{k=1}^{s_j} E\left[X_{j,k}\right]\right] \\
&= \sum_{\substack{j\neq 0\\ j\neq i}} E\left[\sum_{k=1}^{s_j} E\left[X_j\right]\right] \\
&= \sum_{\substack{j\neq 0\\ j\neq i}} E[s_j]E[X_j]
\end{aligned} \tag{A.18}$$

Let us find $E[s_j]$.

$$\begin{aligned}
E[s_j] &= E_L\left[E[s_j|L]\right] \\
&= E\left[(L-1)\frac{p_j}{\sum\limits_{\substack{k\neq 0\\ k\neq i}} p_k}\right] \\
&= E\left[(L-1)\frac{p_j}{1-p_0-p_i}\right] \\
&= \frac{p_j}{1-p_0-p_i}E[L-1] \\
&= \frac{p_j}{1-(p_0+p_i)}\frac{1-(p_0+p_i)}{p_0+p_i} \\
&= \frac{p_j}{p_0+p_i}
\end{aligned} \tag{A.19}$$

where, in the first equality the inner conditional expectation is the expected number of times sensor $j, j\neq i, 0$ is polled given L slots. Finally,

$$E[Z] = \sum_{\substack{j\neq 0\\ j\neq i}} \frac{p_j}{p_0+p_i}E[X_j] \tag{A.20}$$

Since $T_{i,k-1} = X_{i,1} + Z + X_{0,1}^{'}$, therefore,

$$\begin{aligned} E[T_{i,k-1}] &= E[X_i] + E[Z] + E[X_0] \\ &= E[X_i] + E[X_0] + \frac{1}{p_0 + p_i} \sum_{\substack{j \neq 0 \\ j \neq i}} p_j E[X_j] \end{aligned} \tag{A.21}$$